Assesment and Re-Design of VIROMAX

(Self Propelled Kid's Ride)

David Libera	260535474
Ahmed Durrani	260532755
Mikel Garcia-Poulin	260466439
Christina Harvey	260444783

In 1987, the Richer brothers came up with a revolutionary concept for a super swing. It all started with one prototype built in their own backyard and this super swing grew in popularity ever since. Ten years later, in 1997, a new and improved swing was born. This new swing was named VIROMAX. The VIROMAX was then brought to various different events across Québec and was featured in many articles. In June 2000, the owners of VIROMAX were invited to the first *Salon des Inventions du Québec à Montréal*, headed by Mr. Daniel Paquette of the *Inventarium*. At this event, the Richer brothers won the public's choice award and it was a dream come true. VIROMAX is a small company, but they have big dreams and big plans for the future.

"By elevation of 7.5 meters, the attraction VIROMAX offers exceptional performance in addition to creating a dynamic and interactive atmosphere between the participants and spectators. Participants live a unique experience in twirling around an axis while making full rotations of 360 degrees safely."

I. Nomenclature

α	= angular acceleration of secondary shaft about main shaft in (*rad/s*^{2})
σ_a'	= Von Mises Alternating Stress *(ksi)*
σ_b	= Bearing Stress *(ksi)*
σ_i	= Initial Stress*(ksi)*
σ_m'	= Von Mises Midrange Stress *(ksi)*
σ_p	= Proof Stress*(ksi)*
σ_t	= Tensile Stress *(ksi)*
θ_r	= Angle of Rotation of the Main Shaft (*rad*)
τ	= Shear Stress *(ksi)*
ω	= angular velocity of secondary shaft about main shaft in (*rad/s*)
A_b	= Bearing Area *(in^2)*
A_p	= Proof Area *(in^2)*
A_s	= Shear Area *(in^2)*
A_t	= Area in Tension *(in^2)*
A_t	= Tangential reaction force at bearing A (radial force on bearing) *(lbf)*
A_n	= Axial force at bearing A *(lbf)*
B_t	= Tangential reaction force at bearing B (radial force on bearing) *(lbf)*
B_n	= Axial force at bearing B*(lbf)*
C	= Stiffness Constant
C_{cw}	= Centripetal force due to counterweight *(lbf)*
C_b	= Centripetal force due to counterweight *(lbf)*
D	= Diameter of bolts *(in)*
F_{bolt}	= Axial force at bolt *(lbf)*
F_i	= Initial Force (Preload) *(lbf)*
F_{leg}	= Force on the leg *(lbf)*
F_n	= Radial reaction force at press fit *(lbf)*

F_p	= Proof Force *(lbf)*
F_t	= Tangential reaction force at press fit *(lbf)*
K_b	= Bolt Stiffness
K_t	= Stress Concentration Factor
K_f	= Fatigue Stress Concentration Factor
K_{fa}	= Axial Fatigue Stress Concentration Factor
K_{fb}	= Bending Fatigue Stress Concentration Factor
M_r	= Reaction moment at press fit *(lbf-in)*
L_{pin}	= Length of pin with respect to press fit (See Figure 3) *(in)*
L_{ms}	= Length of main shaft with respect to press fit (See Figure 4) *(in)*
n	= Safety factor
P	= External Load *(lbf)*
q	= Notch Sensitvity
r_G	= Distance from press fit to center of mass of cage and person *(in)*
r_b	= Radius of rotation of the child and cage (w.r.t. the center of gravity) *(in)*
r_{cw}	= Radius of rotation of the counterweight (w.r.t. the center of gravity) *(in)*
R_{A_y}	= Reaction force from main shaft in y direction, applied at edge of press fit *(lbf)*
R_{A_x}	= Reaction force from main shaft in x direction, applied at edge of press fit *(lbf)*
R_{Bo_y}	= Reaction force from the pin's outer bearing in y direction, applied at z_{Bo} *(lbf)*
R_{Bo_x}	= Reaction force from the pin's outer bearing in x direction, applied at z_{Bo} *(lbf)*
R_{Bi_y}	= Reaction force from the pin's inner bearing in y direction, applied at z_{Bi} *(lbf)*
R_{Bi_x}	= Reaction force from the pin's inner bearing in x direction, applied at z_{Bi} *(lbf)*
S_e	= Endurance Limit *(ksi)*
S_{sy}	= Static Yield Strength *(ksi)*
S_p	= Proof Strength *(ksi)*
S_y	= Yield Strength *(ksi)*
$S_{y,\ members}$	= Member Yield Strength *(ksi)*
S_{ut}	= Ultimate Tensile Stress *(ksi)*
W_{ms}	= Weight of Main Shaft, applied at Half Span *(lbf)*
W_{nut}	= Weight of nut, applied at z_{nut} *(lbf)*
W_{pin}	= Weight of pin, applied at L_{pin} *(lbf)*
W_{cw}	= Weight of Counterweight, half load applied at z_{cw1} and z_{cw2} *(lbf)*
z_{Bi}	= Location of center of the pin's inner bearing with respect to press fit (z=0) *(in)*
z_{Bo}	= Location of center of the pin's outer bearing with respect to press fit (z=0) *(in)*
z_{nut}	= Location of center of the pin's nut with respect to press fit (z=0) *(in)*
z_{cw1}	= Location of first attachment of the counterweight to the main shaft *(in)*
z_{cw2}	= Location of second attachment of the counterweight to the main shaft *(in)*

II. Table of Contents

III. Introduction

This report summarizes the fatigue analysis of the critical components of the VIROMAX, specifically the main shaft, secondary shaft, bolted joints and bearings. Based on the results components were re-designed and the final configuration of the ride will be presented. Our task is to assess the overdesigned components and re-design (if necessary) the self-propelled kid's ride VIROMAX. The goal is to evaluate the safety factors of the main components to see where we can save weight. The main components that need assessment are: 1) the main shaft (1^{st} axis of rotation), 2) secondary shaft (2^{nd} axis of rotation), 3) bolted joints, 4) bearings. VIROMAX has given some information regarding the components above. The maximum velocity recorded of the main shaft is 60 RPM. The maximum velocity of the second shaft is 150 RPM. The maximum weight considered is 300 lbm. Operating temperature is from -40°C to +40°C. In addition, a Bill of Materials was provided along with some 2D CAD drawings and a 3D SolidWorks assembly. Lastly, we were told to assume infinite life and we were given a reliability factor of 99%.

A. Axis Definitions

The figures below for the appropriate axis and start points used during the analysis and referenced through the report.

Figure 1 - VIROMAX Front View

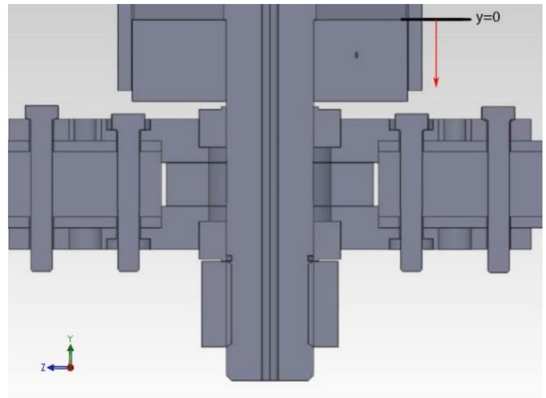


Figure 2 – Secondary Shaft Definition of 0

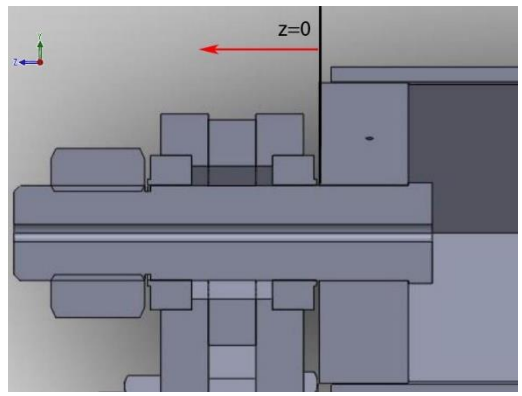


Figure 3 - Pin Definition of 0

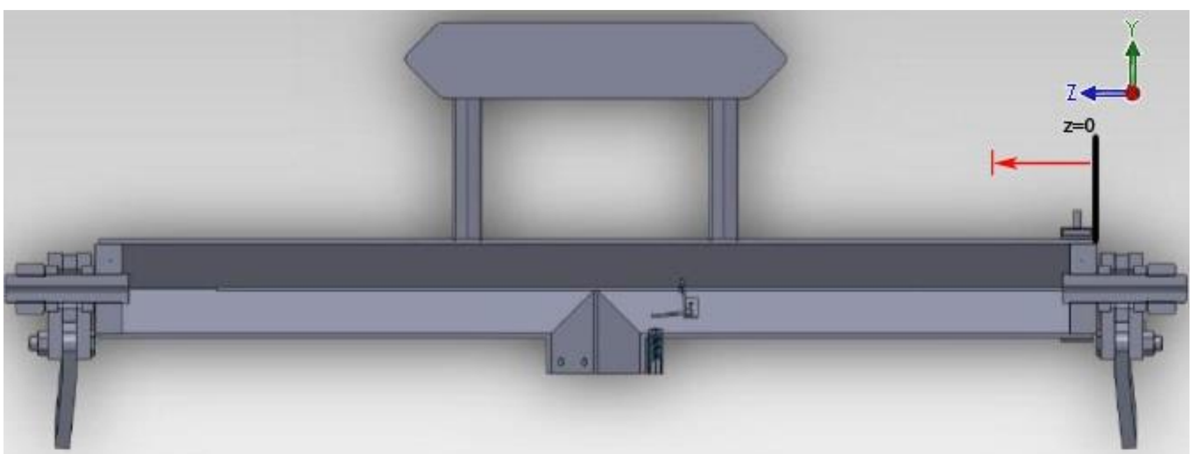


Figure 4 – Main Shaft Definition of 0

B. Assumptions and Justifications

The following list includes the main assumptions used while deriving the system of equations and analysis of the swing. Any further or very specific assumptions will be outlined as they are implemented within the analysis approach.

- It was assumed that the pin at the ends of the main shaft were in fact acting as a shaft and the applicable analysis was carried out on those components. This report refers to this shaft as a "pin".
- The reaction forces on either side of the main shaft were equivalent therefore only one side of the swing was analysed.
- Neglect weight and dynamics of the braking system i.e. this analysis is only applicable to a swing configuration that does not have a brake.
- A human body can only handle a vertical (human body frame of reference) G force of approximately 5 G. The maximum RPM of the main shaft given by the project was 60RPM and this results in a 7.25G which would only be useful for a ride if trained fighter pilots were enjoying it. Standard rollercoasters allow 5-6Gto be maintained for 1 second therefore a maintained force of 7.25G would in fact result in the death of a rider. Additionally, as the swing is self propelled it would be impossible for a rider to reach this maximum RPM prior to fainting. Therefore the maximum steady state speed of this apparatus was decreased to 5G (approximately 50RPM). To this effect the analysis of the swing was performed for both the maximum load of 60 RPM and the 50 RPM. However, due to the human factor, we designed our components for a load of 50RPM as it was concluded that 60 RPM was an impossible and potentially fatal rotation speed.
- Assume the center of gravity of the rider to be perfectly centered underneath the center shaft in all directions, resulting in a perfectly symmetric cage and load.
- Due to the previous assumption, it was assumed that the main components were not experiencing any axial forces (i.e. along the z axis).
- The torque on the main shaft was assumed to be in phase with the moment to simplify calculations and provide a conservative estimate.
- Equivalently the axial force and bending moment on the secondary shaft was also assumed to be in phase.
- Bolts are temperature and weather resistant
- Physical properties are assumed to be constant
- Yield strength, proof strength, etc are all based off literature values
- Fastners have no defect (internally or externally)
- No eccentric loading, only simple loading cases
- Perfect symmetry about the center axis of the shaft which implied the centripetal forces from equal masses about the axis cancel
- At the secondary shaft, the radial and tangential stresses, naturally out of phase by 90 degrees were put in phase and divided by $\sqrt{2}$ in order to perform the fatigue analysis and accommodate for the unrealistically higher resultant stress
- The bearing radial reaction forces were assumed zero and the bolt at the bottom took the entire radial load. A free body diagram of the assembly in Appendix B was found to be statically indeterminate.
- The shear stress in the shaft were not analyzed due to the more critical nature of the combined bending and axial stresses

IV. MATLAB Codes and Inputs

The following codes were used throughout the analysis of the VIROMAX and the full codes may be seen in AppendixX. These codes will be referenced throughout the report and are therefore clarified at the beginning to simplicity.

Table 1. MATLAB Codes

Code Name	Inputs	Outputs
Bearing_tapered	-Magnitudeof Radial Force Inner Bearing -Magnitudeof Radial Force Outer Bearing -External Axial force -Pin Or Secondary Shaft choice	-Ratio of max force in catalogue equivalent radial force applied
Slope_cantilevered_and_simple_moment	-Reaction Force -Inner diameter of shaft -Outer diameter of shaft	-Max slope
Slope_cantilevered	-Intermediate Force -Outer Force -Inner diameter of shaft -Outer diameter of shaft	-Max slope
Calculate_Se	- S_{ut} -Load Type (bending/axial/torsion) -Surface Condition -Cross-Section Shape -diameter/outer width/inner width	-S_e
fatigue_safety_factor_mod_goodman	-S_e - S_{ut} -σ_a' -σ_m'	-$SF_{fatigue}$
FBD1_Sec_Shaft	-Rn - Rt -Mr -lengths to bearing forces (l1,l2)	-Forces at bearings (At, Bt, Fbolt)
little_shaft_dynamics (Pin)	-Weight of pin -Weight of large nut -Reaction force from Main Shaft -Location of bearings -Location of nuts -Length of shaft -Location of point to evaluate	-Alternating Moment -Midrange Moment -Reaction forces on bearings
main_shaft_dynamics	-Weight of Main Shaft -Weight of Counterweight -Weight of Child+Cage -Location of Counterweight attachment -Radius of Center of Gravity for Child+Cage -Radius of Center of Gravity for Counterweight	-Alternating Moment -Midrange Moment -Alternating Torsion -Midrange Torsion -Reaction forces on the Pin
Master Code	-location of point to evaluate (x)	-Prints Fatigue Analysis Parameters and Plots
Main_Script	N/R	-Evaluates all Results on the Pin and Main Shaft
MaxMinMid	-Axial, Bending and Torisonal Stresses	-σ_{axial_alt} -σ_{axial_mid} $\sigma_{bending\ alt}$ -$\sigma_{bending\ mid}$ -τ_{alt} -τ_{mid}

safety_factor_static	-S_y -σ'_a -σ'_m	-SF_{yield}
stress_cylinder	-Inner and Outer Diameter -Axial Force -Bending Moment -Torsion	-σ_{axial} -$\sigma_{bending}$ -τ
Stress_Sec_Shaft1	-Forces acting on shaft (At, Bt, Rn, Rt, Mr) -Geometry (l1,l2,l3,Area) -location of point to evaluate (x)	-Axial, Bending and Torsion Stresses at location x (SAx, SBx, STx) -Moment (Mx)
stress_thin_wall_square_shaft	-Inner and Outer Width -Axial Force -Bending Moment -Torsion	-σ_{axial} -$\sigma_{bending}$ -τ
Rxn_Forces	- angular speed (w) -mass of cage and person (m)	-Reaction Forces and Moment (Rn, Rt, Mr) -time
von_mises_stresses	-σ'_a -σ'_m	-σ_{axial_alt} -σ_{axial_mid} $\sigma_{bending\ alt}$ -$\sigma_{bending\ mid}$ -τ_{alt} -τ_{mid}

V. Analysis Approach

The materials used in the design of the VIROMAX and their specific properties as used for the analysis are listed in Table 1.

Table 2. Material Properties

Material	Short Name	Density (lb/m^3)	S_{ut} (ksi)	Sy (ksi)
Stainless Steel	SS304	0.2890	-	-
Carbon Steel	C12L14	0.2840	78.3	60.2
Aluminum	6061-T6	0.0975	-	-
Structural Steel	G40.21	0.2836	65.0	50.0
Mild Carbon Steel	Gr.8.8	0.2840	120.0	95.7
Bronze	Bronze	0.3100	-	-
LUX-R Foam	Foam	0.0010	-	-
Steel	ASTM A-283 Gr. D	0.2840	67.5	33.0

A. Main Shaft

The design of the main shaft was developed by first evaluating the static free body diagrams for the swing. We then built on these designs to add in the dynamic forces due to centripetal force. Due to the redesign process it was decided to fully hardcode our dynamic free body diagrams in a way that we could simply import values (such as critical weights and measurements) froma master excel document. Re-running the code would allow us to minimize calculation time.

1. *Static Analysis*
   The first codes built were the static forces which yielded the moment diagram seen in Figure 5 where z=0 was taken to be the left side of the swing when seen from the front view (Figure 1). As expected for our static analysis we see the largest magnitude moment at the center of the main shaft.

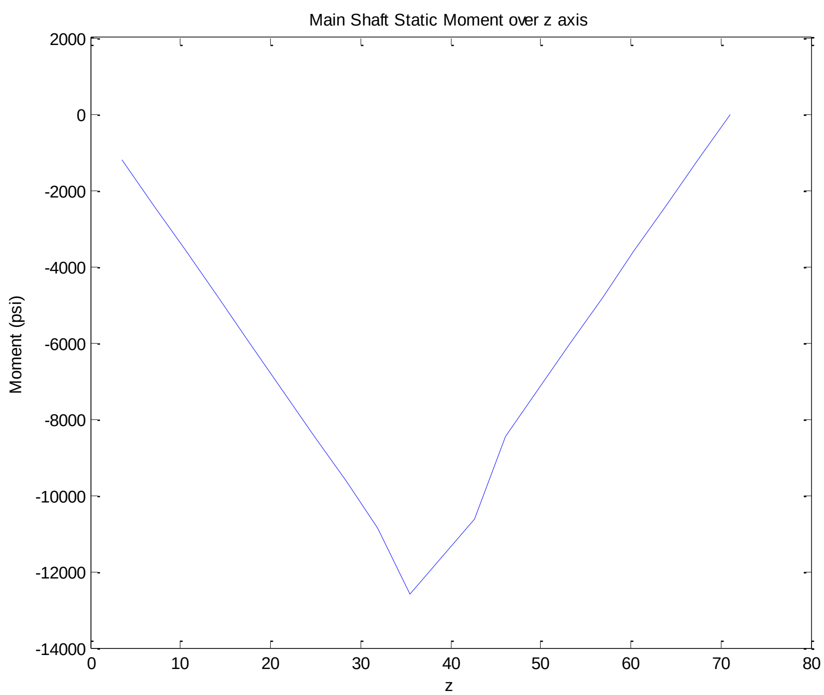


Figure 5 – Static Moment Diagram - Main Shaft

2. *Dynamic Analysis:*
   For our dynamic analysis of the main shaft we built a very similar code (main_shaft_dynamics) to that for static however it was amended to include dynamic forces. We derived the moment equations (Equations 1 and 2) for the moment about the x and y axis. This particular form of the equation is only valid if the axial distance, z, is between the end of the shaft (the right side according to the front view Figure 1) and the second counterweight attachment point. Our moment MATLAB code modifies the moment formula accordingly depending on the exact location desired on the shaft. Additionally, it was discovered that the counterweight would create a completely reversed torque on our main shaft as seen by Equation 3. This was a result of the weights creating opposing torques about the z axis

when the swing is at either 90º or 270º. These equations are determined from the free body diagrams that can be seen in Appendix X.

$$M_x = -R_{A\_y}z + \left(\frac{W_{cw}}{2}\right)(z - z_{cw1}) + (C_{cw} - C_b)\cos\theta_r \left(z - {}^{L_{ms}}/_{2}\right) + (W_b + W_{ms})\left(z - {}^{L_{ms}}/_{2}\right) + \left(\frac{W_{cw}}{2}\right)(z - z_{cw2}) \quad (1)$$

$$M_y = R_{A\_x}z + (C_{cw} - C_b)(\mathrm{z} - \mathrm{L_{ms}}/2)\sin\theta_r \quad (2)$$

$$T = (W_b r_b - W_{cw} r_{cw})\sin\theta_r \quad (3)$$

Using these equations we iterated through 0º to 360º over the full length of our shaft such that for each location it was possible to determine the alternating and midrange moments and/or torques.

The next step in evaluating the main shaft was to pin point the critical locations that would be the largest stress risers. The stress concentrations factors were chosen to be extremely conservative to allow for the safest and most effective evaluation of the shaft. All factors were determined from the reference literature. For CP3 no exact values could be found for the configuration instead a value was used as if the shaft had a vertical hole all the way through the shaft. Therefore this factor was much larger but at least provided an extremely conservative result. The notch sensitivity, q, was conservatively chosen to be 0.8 at all locations.

Table 3. Critical Points on the Main Shaft

Critical Point	**Location**	**$K_{t\ bending}$**	**$K_{t\ torsion}$**
CP1	Connection of the bird beak SHS T joint *(Filleted location)*	1.8	1
CP2	Inner Corner at Half Span *(Location of the largest moment)*	1	1.25
CP3	Hole on edge of Shaft *(For emergency braking system)*	5	3.5

B. Pin

The following equations were derived from the free body diagrams on the pin, which are found in the appendix. All z forces are neglected. Torque on the pin is assumed to be zero because of the perfect press fit assumption and zero friction on the bearings.

$$\sum F_z = 0$$
$$\sum F_y = -R_{A\_y} + R_{Bo\_y} + R_{Bi\_y} - W_{nut} - W_{pin} = 0$$
$$\sum F_x = -R_{A_x} + R_{Bo\_x} + R_{Bi\_x} = 0$$

$$\sum M_{z@a} = 0$$
$$\sum M_{y@a} = R_{Box} \times z_{Bo} + R_{Bix} \times z_{Bi} = 0$$
$$\sum M_{x@a} = -R_{Boy} \times z_{Bo} - R_{Biy} \times z_{Bi} + W_{nut} \times z_{nut} + W_{pin} \times \frac{L_{pin}}{2} = 0$$

The pin connected to the main shaft is modelled as a shaft. This is done because this part spins on two bearings, and it therefore facilitates the analysis required to find the resultant forces on the bearings. The pin is assumed press fit into the square inner sleeve, which is then connected to the main shaft via fasteners. The fasteners and the press fit are not analyzed in this report and assumed perfect to simplify the model.

Assumptions:
1. Assume perfect press fit between pin and inner square sleeve.
2. Assume perfect fasteners between main shaft and inner square sleeve.
3. Neglect all axial forces in pin due to main shaft or legs.
4. Neglect forces on sleeve between both bearings, since axial forces are neglected.
5. Neglect deflection and slope in pin. This assumption is validated quantitatively later on.
6. Assume no friction forces in bearings.
7. Assume no torsion on pin . Assume that all torsional stress on main shaft is taken by the main shaft or the press fit, and pin is perfectly free to turn on bearings without torsion.
8. Assume pin begins at edge of press fit.
9. Assume point forces at beginning of pin, center of bearings and center of nut.
10. Weight of pin is modeled as a point force applied at center of mass of shaft.
11. Assume q=0.8 for a conservative multiplier of the stress concentration factor.
12. Neglect weight of bearings due to significant reaction forces at bearing much larger than their weights.

The Free Body diagram for this part is located in Appendix B.

.

The stress concentration factors for the pin can be seen in Table 4. These factors are estimations found in Shigley's text the main discrepancy between these values and those in the text are that they do not take into account the hole through the center of he shaft. Due to the small inner diameter it was assumed that these approximations were still valid and were each increased by 0.1 to attempt to account for potentially larger stresses caused by the inner hole. As can be seen by the results this make sense as even if a larger number was used the pin would still have very large safety factors.

Table 4. Critical Points on the Pin

Critical Point	Location	$K_{t\ bending}$	$K_{t\ torsion}$
CP4	Small diameter change – closest to inner bearng *(Filleted location)*	2	N/R
CP5	Center of Inner Bearing Seat *(Location of the largest moment)*	1	N/R
CP6	Large diameter change after the outer bearing *(Location of smallest diameter)*	2.6	N/R

C. Secondary Shaft

A static analysis and a failure analysis were performed on the secondary shaft to ensure its design was reliable and safe. The following report is a documentation of the procedure taken throughout the analysis including free body diagrams, equations of motion, assumptions, approaches and methods applied.

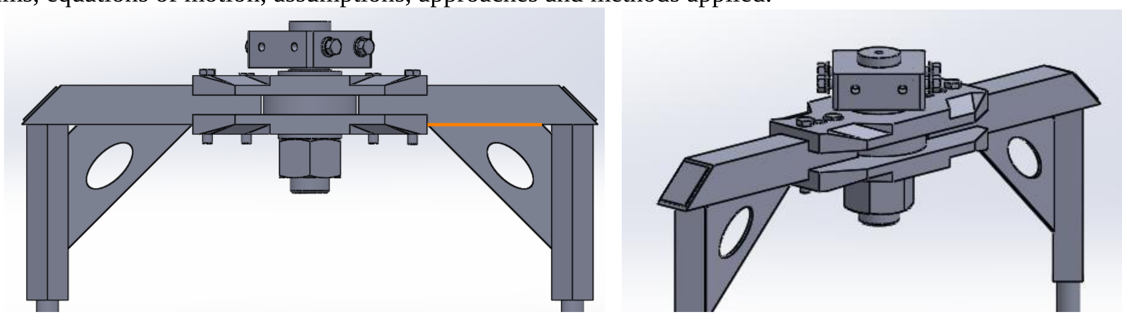


Figure 6 - Views of Secondary Shaft

1. Static Analysis

First, the loading for the secondary shaft under stationary conditions was considered. This consisted of a simple application of the static safety factor equation for the stress induced by the weight of the cage and 300 lbm person. To understand the loading for the fatigue analysis, the reaction forces were derived for a rigid body rotating about a fixed axis. The analysis is described in detail in the next section.

2. Dynamic Analysis

The steady state behavior of the secondary shaft was analyzed for a constant rotational angular velocity, simplifying the analysis similar to the main shaft. Firstly, the reaction forces at the shaft's end, where it connected to the main horizontal shaft, needed to be found. The free body diagram in Figure 7 and the graphs in Figure 8 demonstrate the fluctuating nature of the reaction forces and moment due to the weight and centripetal force. It is important to note that the reaction radial forces is out of phase by 90 degrees with the tangential force and moment. Fatigue analysis using the Modified Goodman criteria requires the stresses to be in phase. In order to accommodate for this, the force was set in phase and all force and moment magnitudes were divided by $\sqrt{2}$. The formal approach to out-of-phase loads was never formally studied in class. With the equilibrium equations about a fixed axis in the normal and tangential components, the reaction forces at the press fit could be solved for. For steady state rotation, the analysis is simplified by assuming constant angular velocity and zero angular acceleration ($\omega \neq 0$ & $\alpha = 0$).

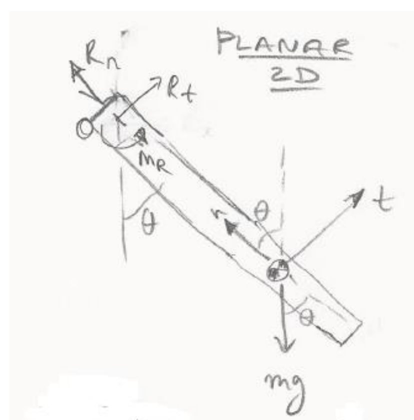


Figure 7 – 2D FBD of Secondary Shaft

$$\sum F_n = m\omega^2 r_G = F_n - mgcos\theta \quad \rightarrow \quad F_n = m\omega^2 r_G + mgcos\theta \quad \rightarrow \quad F_n = m\omega^2 r_G + mgsin\theta$$

$$\sum F_t = m\alpha r_G = F_t - mgsin\theta \quad \rightarrow \quad F_t = mgsin\theta$$

$$\sum M_\theta = I_o\alpha = mgr_G sin\theta - M_r \quad \rightarrow \quad M_r = mgr_G sin\theta$$

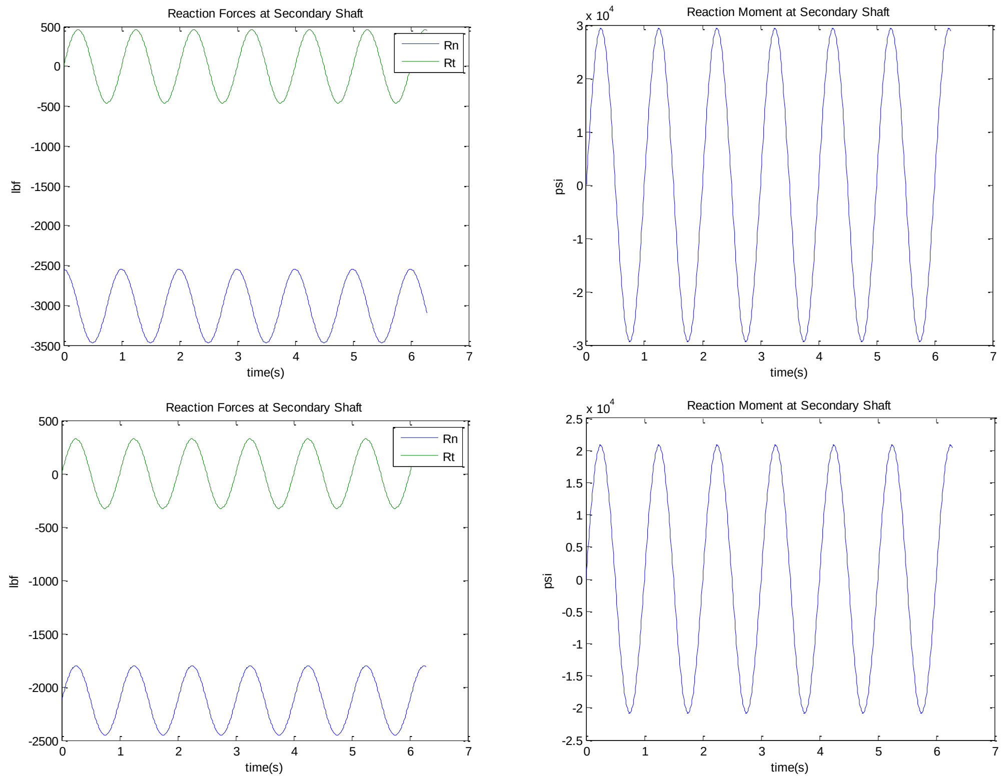


Figure 8 – Actual Reaction Forces and Moment (top row), Modified forces and moments (bottom row)

Next, a free body diagram of the secondary shaft along with its equilibrium equations would determine the forces acting on the shaft. It was derived by pulling apart the entire assembly and observing the reacting forces and their effect on one another. With an excess amount of forces and limited equilibrium equations, the assembly was deemed statically indeterminate in the vertical direction. The figure entitled *secondary shaft* in appendix demonstrates the excess amount of unknown forces along the *t*-axis. The shaft vertical reaction forces at the bearings (An, Bn) had to be neglected.

The free body diagram in Figure 9 illustrates the forces considered for the analysis. The assumption made the axial force constant throughout the shaft. It can be seen, in figure 9, that the bolt and bottom plate exert axial forces on the bottom bearing. This would be important to apply design bearing with this load in mind. The top bearing can be assumed to experience no axial load. The tangential forces (Rt, At, Bt) caused a variation in bending moment. The moment diagram for the loading case is illustrated in figure 10 for the modified forces mentioned earlier.

Also, the rotation of the shaft was considered for the case when the person is standing direction below the shaft. It was determined that the centripetal forces from the masses offset from the central axis of the shaft would cancel out if the cage is considered symmetrical. Thus, it was assumed that there was perfect symmetry about the secondary shaft and no torsion was induced on the shaft.

With the moment diagram and axial load at every point in the shaft, the stresses, alternating and midrange, at points on the outer surface of the shaft could easily be solved for. It was established that there were 3 locations of interest to analyze for safety factors at the outer surface of the shaft. Their respective stress concentration values are listed in table 5. Usually, stress concentration factors are modified by a notch sensitivity value for conservative values but it was decided to assume the shaft had no notch sensitivity for this preliminary redesign. For a more thorough test, this would not be the case.

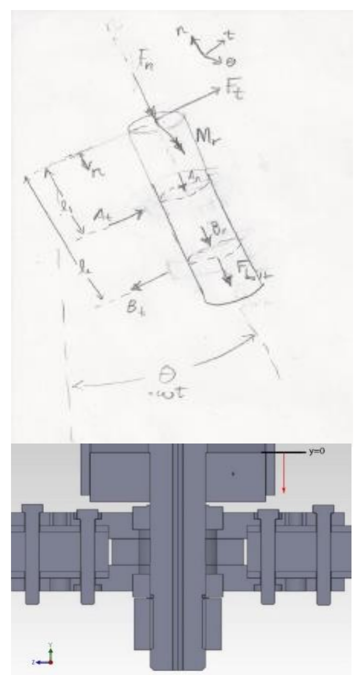


Figure 9 – FBD of Secondary Shaft

Table 5. Critical Points on the Secondary Shaft

Critical Point	**Location**	**$K_{f\ bending}$**	**$K_{f\ torsion}$**
Press Fit	Shaft press fit *(Location of largest moment)*	1	1
Bearing A	Inner bearing location with stress concentration *(Location of the very large moment)*	1.05	1.05
Bearing B	Outer bearing with largest stress concentration *(Keyway)*	4.5	4.5
Bolt	Large bolt holding the weight of the person and cage (*As a reference value*)	1	1

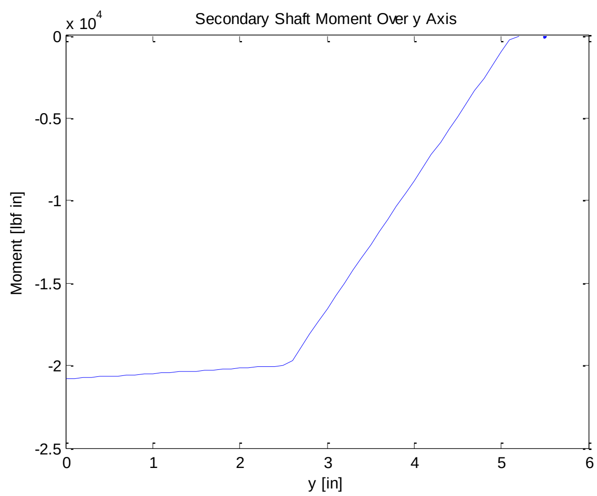


Figure 10 –Moment diagram for maximum load case

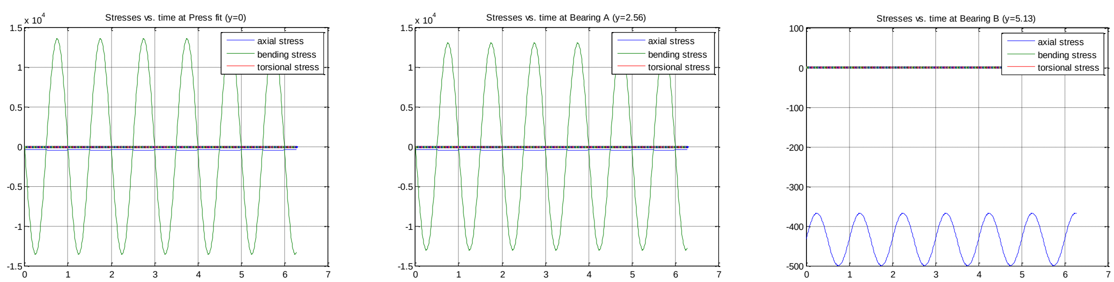


Figure 11 – Stresses vs. Time for press fit (left), bearing A (center) and bearing B (right) for maximum load case

It can be seen from figure 11 that the bending stresses dominate for the press fit and bearing A while the axial load is the main source of loading for bearing B since it is the free end of the shaft and consequently experiences no bending moment.

D. Bearings

1. *Bearing Analysis*

C_{90} is a Timken designation, assumed equivalent to C_{10} except for a life of 90x10^6 cycles.

$$C_{90} = a_f F_{max}\left[\frac{x_D}{x_0 + \left[(\theta - x_\theta)\ln\left(\frac{1}{R_d}\right)\right]^{\frac{1}{b}}}\right]^{\frac{1}{a}}$$

X_0, Ɵ and b are Weibull parameters for a tapered roller bearing.
R_d = Reliablity
a = 10/3 for tapered roller bearing
F_{max}= equivalent radial force
a_f = application factor (assumed = 1)

Equations for two tapered roller bearings mounted next to each other are shown below (taken from Shigley). Bearing A is the bearing that takes the axial load and the induced axial load from bearing B. The induced axial load is due to a radial load , therefore an induced axial load will exist without at an external axial load.

$$Fe_A = 0.4F_{rA} + K_A\left(\frac{0.47F_{rB}}{K_B} + F_{a\_ext}\right)$$
$$F_{eB} = F_{rB}$$

There are two bearings located on each pin and two bearings on the secondary shaft. Though axial forces are ignored on the pin, they are not on the secondary shaft. The Bill of Materials provided indicates 6 identical Conical Roller bearings (or Tapered Roller Bearings) were used in the original design. Since the bearings are not explicitly described in the Bill of Materials, tapered roller bearings fitting the dimensions from the Solidworks CAD assembly were chosen from the catalogues online, in the lecture notes and in the textbook. Tapered roller bearings for almost the exact dimensions were found in a Timken Tapered roller bearing catalogue online (link provided references section).

The 6 identical bearings assumed to be in the initial design are:

Bore d	O.D. D	Width T	Dynamic[1] C_1	Factors[2] e	Y	Dynamic[3] C_{90}	C_{a90}	Factors[2] K	Static C_0	Inner	Outer
mm in.	mm in.	mm in.	N lbf			N lbf	N lbf		N lbf		
50.800 2.0000	82.931 3.2650	21.590 0.8500	96300 21700	0.31	1.97	25000 5610	13000 2930	1.91	104000 23400	LM104949	LM104912

Figure 12 – Timken Catalogue for bearing results

To calculate an approximated safety factors on the bearings, the maximum load on the bearing for infinite life (with XD = 10^6/(90x10^6) = 1/90) were isolated and calculated for using the C_{90} reference in the catalogue. This maximum force was then compared to the effective radial force applied on the shaft. An individual bearing reliability of 99.5% was used to achieve the target of 99% reliability for the bearing setup. Tapered roller bearings induce axial forces on each other when under radial loads. Therefore it is necessary to calculated equivalent radial loads. The equivalent radial forces were calculated using external axial loads, external radial loads and induced axial loads (from the bearing not taking any axial force) on the bearing taking the external axial force .

Secondary Shaft Bearings
The external bearing is assumed take the axial loads for the secondary shaft setup since the shaft does not spin but the housing around it does.

Pin Bearings

The inner bearing is assumed to take the induced axial load from the other bearing in the pin setup. Since there is no axial force on the pin it is not clear which bearing takes the induced load therefore it is assumed that the critical bearing takes the load to keep the safety factors conservative.

This methodology was coded into a Matlab function which therefore solved for the ratio of maximu radial load (solved by C_{90} equation) to the equivalent radial load applied. This was done for both bearings on both the pin and the secondary shaft.

2. *Pin Slope Analysis*

Equations for deflections were taken from Shigley's Mechanical Engineering Design 9th Edition and then differentiated to get the following slope equations. This analysis was on the pin and the secondary shaft to make sure the bearings do not experience too great of a slope. A simply supported beam with a moment is added to a cantilevered beam with point force on the end to model the pin. Simple supports represent the bearing reaction forces while the reaction force from the main shaft is represented as a point force. A cantileverd beam with two points forces is used to model the secondary shafts setup, with cantilevered joint being the pressfit between the secondary shaft and the main shaft, and the point forces the forces on the bearings.

Simply Supported Beam with one point force and a moment (and b =0, i.e moment at second support):

$$slope_{simply\ supported\ with\ moment} = \frac{M_B}{6EIL}(x^2 + 3a^2 - 6aL + 2L^2) + \frac{M_B x}{6EIL}(2x)$$

Cantilevered Beam with point force at end:

$$slope_{cantilever} = \frac{2Fx}{6EI}(x - 3L) + \frac{Fx^2}{6EI}(1)$$

Summing the two:

$$slope = \frac{M_B}{6EIL}(x^2 + 3a^2 - 6aL + 2L^2) + \frac{M_B x}{6EIL}(2x) + \frac{2Fx}{6EI}(x - 3L) + \frac{Fx^2}{6EI}(1)$$

Cantilevered Beam with 2 point forces:

F_i = Intermediate Force applied
F_o= Outer Force applied (at end of beam)
a = distance from zero to intermediate force
b= distance from intermediate force to end of beam
I= moment of inertia of beam
L= length of beam

$$\theta_{AB} = \frac{2F_i x}{6EI}(x - 3a) + \frac{F_i x^2}{6EI} + \frac{2F_o x}{6EI}(x - 3L) + \frac{F_o x^2}{6EI}$$

$$\theta_{BC} = \frac{F_i a^2}{6EI}(-3) + \frac{F_o x^2}{6EI} + \frac{2F_o x}{6EI}(x - 3L)$$

Slopes	
Tapered roller	0.0005–0.0012 rad

Figure 13 – Maximum Slopes of Tapered Roller Bearings (Budynas & Nisbett, 2011)

As can be seen in the above table taken from Shigley (page 379, 9th Edition), depending on the particular tapered roller bearing the maximum slope allowed is anywhere between 0.0005 rad and 0.0012 rad.

To verify this two Matlab codes were created. This Matlab code used the equations above to calculate the maximum slope on the whole shaft or pin. Moments of inertia were calculated internaly in the code.

Table 6. Bearing Maximum Slopes

	Maximum Slopes	
	Pin	*Secondary Shaft*
60 RPM	3.719×10^{-4} rad	5.768×10^{-4} rad
50 RPM	2.783×10^{-4} rad	5.768×10^{-4} rad

As can be seen the results are all significantly under 0.0012 rad. The pin is safely under 0.0005 rad while the secondary shaft is around that region but slightly over. These slopes occur at unrealistic loads therefore it is assumed that slopes do not play a large role in the analysis of the initial design, and tapered roller bearings can be safely used anywhere on the shafts.

E. Bolted Joints

The analysis of the fasteners is an important aspect of any assembly design. They hold components together, experiencing fluctuating tension and shear forces which needed to be sustained. From literature, it is known that the ideal bolt length is one where only two threads project from the nut after it is tightened. Bolt holes can usually have differences in geometry which lead to unanticipated stress concentrations. Therefore, washers are used under the head of the bolt to prevent this. Since the function of the bolt is to hold or clamp pieces together, the clamping load stretches or elongates the bolt. This is obtained by tightening the bolt until just before its elastic limit. The tightening force is knows as the preload. During tightening, the first thread of the nut takes the entire load but yielding occurs and divides this force over 3-4 threads. Because of this, it is advised never to reuse nuts.

Below, the locations can be seen for the different types of fasteners. Their respective free body diagrams are located in the appendix. These forces will be taken into account when calculating their respective safety factors.

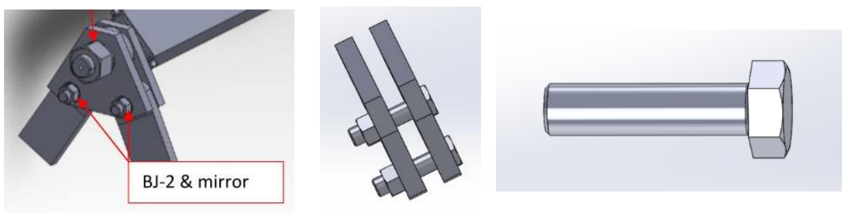


Figure 14 – BJ2

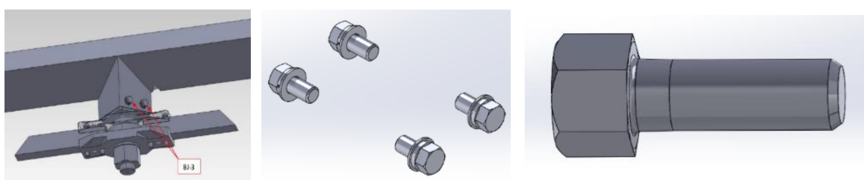


Figure 15 – BJ3

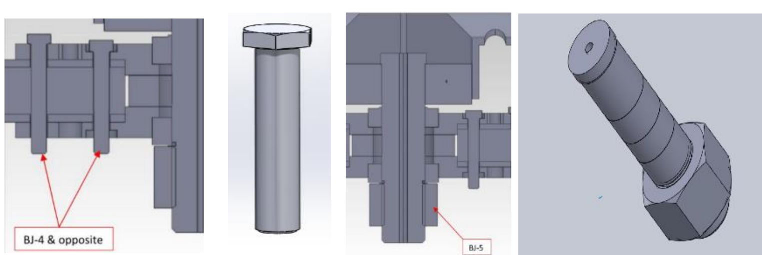


Figure 16 – BJ4 (left) and BJ5(right)

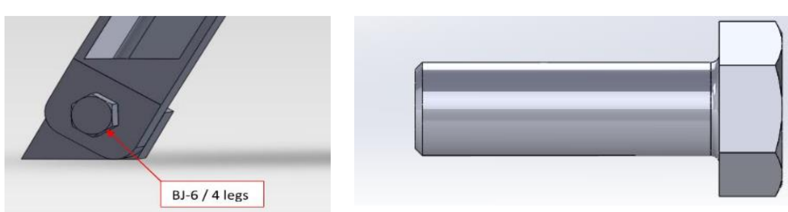


Figure 17 – BJ-6

Sample Calculations for Bolts in Shear.

Please note: this procedure applies for all bolts in shear; only the values will change

Shear Area:

$$A_s = \frac{\pi D^2}{4} = \frac{\pi(1)^2}{4} = 0.7854\ in^2$$

Force in the legs:

$$F_{Leg} = 1747.1 \quad \ldots\ @50RPM$$

Finding resultant Shear Force on Bolt through FBD:

$$\sum F_y = 0 = F_{Leg}\frac{\cos\theta}{4} - R_y\ \therefore\ R_y = 1747.1$$

Shear Stress:

$$\tau = \frac{F_{Shear}}{A_{shear}} = \frac{\frac{F_{Leg}cos\theta}{4}}{0.7854} = \frac{F_{Leg}}{3.154} = 553.93$$

Safety Factor by shear on bolts:

$$n = \frac{S_{sy}}{\tau} = \frac{55.23}{\frac{F_{Leg}}{3.154}} = \frac{174.195}{F_{Leg}} = 9.9705$$

Bearing Area:

$$A_b = length\ x\ width = 1x1 = 1in^2$$

Bearing Stress:

$$\sigma_b = \frac{-\frac{F_{Leg}cos\theta}{4}}{A_b} = -\frac{F_{Leg}}{4.016} = 435.035$$

Safety factor of bearings on bolts:

$$n = \frac{S_y}{\sigma_b} = \frac{95.72}{\left|-\frac{F_{Leg}}{4.016}\right|} = \frac{384.42}{F_{Leg}} = 22.003$$

Safety Factor of bearings on members:

$$n = \frac{S_{y_{members}}}{\sigma_b} = \frac{60.19}{\left|-\frac{F_{Leg}}{4.016}\right|} = \frac{241.723}{F_{Leg}} = 13.836$$

Area in tension:

$$A_t = \frac{[4.62 - (3.16 - 1)]}{4} = 0.615$$

Tensile stress:

$$\sigma_t = \frac{\frac{F_{Leg}cos\theta}{4}}{A_t} = \frac{F_{Leg}}{2.4699} = 707.356$$

Safety factor from tension:

$$n = \frac{S_{y_{members}}}{\sigma_t} = \frac{60.19}{\left|\frac{F_{Leg}}{2.4699}\right|} = \frac{148.60}{F_{Leg}} = 8.506$$

Sample Calculations for Fatigue Loading of tension joins:

Proof force:

$$F_p = \sigma_p * A_p = 600 * \frac{\pi D^2}{4} = 87.022 * \frac{\pi 0.5^2}{4} = 17.086\ KSlugs$$

Initial force (preload):

$$F_i = (0.75)F_p = (0.75) * 17.086 = 12.815$$

Bolt Stiffness:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}\ ;\ k_m = \frac{0.5774\pi ED}{2\ln(5\frac{0.5774l + 0.5d}{0.5774l + 2.5d})}$$

Stiffness Constant C:

$$C = \frac{k_b}{k_b + k_m} = 0.517$$

Factor of Safety against bolt failure:

$$n = \frac{S_p A_t - F_i}{CP} = \frac{87.022 * (0.5 * 0.5) - 12.815}{0.517 * 1.41} = \mathbf{12.26}$$

Factor of Safety against bolt separation:

$$n_o = \frac{F_i}{P(1-C)} = \frac{12.815}{1.41(1-0.517)} = \mathbf{18.82}$$

For fatigue loading of Tension joints, the following sample calculations are provided:

$$S_a = \frac{S_e \sigma_a (S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)}$$

Fatigue factor of safety

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} = \frac{18.709(120.38 - 65.26)}{120.38 * 0.339 + 18.709(0.339 - 65.26)} = 0.\mathbf{8785}$$

Fatigue factor of safety using Goodman criteria:

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{18.709(120.38 - 65.26)}{2.996(18.709 + 120.38)} = \mathbf{2.475}$$

Fatigue factor of safety using Gerber criteria:

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e - \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$
$$= \frac{1}{2 * 2.996 * 18.709}\left[120.38 * \sqrt{120.38^2 + 4 * 18.709(18.709 - 65.26)} - 120.38^2 - 2 * 65.26 * 18.709\right] = \mathbf{38.386}$$

Fatigue factor of safety using ASME-elliptic criteria:

$$n_f = \frac{S_e}{\sigma_a(S_p^2 + S_e^2)}(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e = \frac{18.709 * 4046.197}{23735.83} = \mathbf{3.189}$$

VI. Results

A. Main Shaft

Using the Main Script we ran each of the critical points to calculate the moment, torsion, Von Mises Stresses and using these results and the Modified Goodman equation for infinite life we found the Fatigue Safety Factor. Additionally, we calculated the static yield safety factor by combining the alternating and midrange Von Mises Stress at each location. The results for each point can be seen in Table 7. Figure 18 displays the results found from running the Main script and determining the alternating and midrange stresses at each location over the Main Shaft. These results are for the maximum load case of 60 RPM Main Shaft rotation and a 300lb rider.

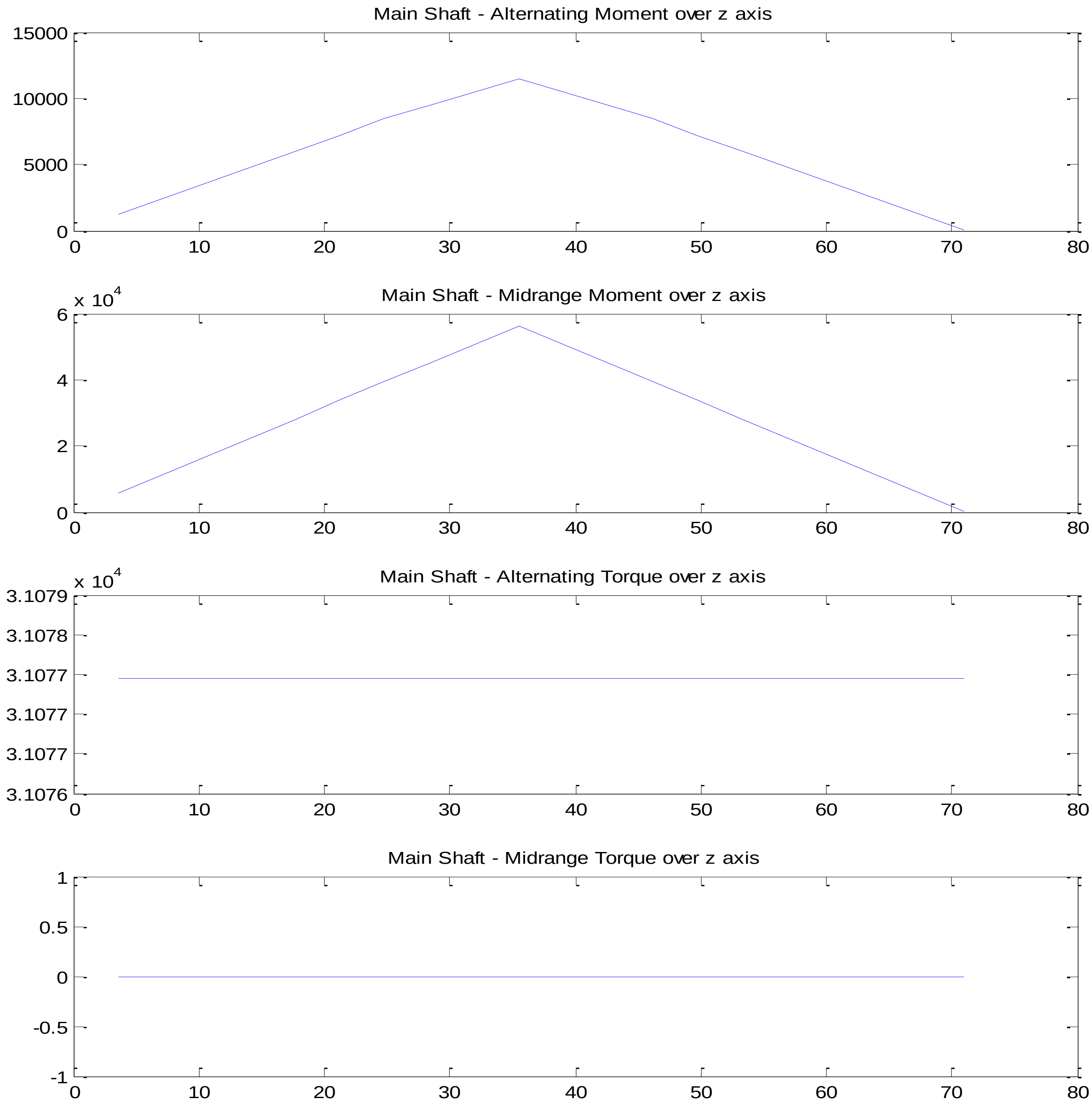


Figure 18 – Dynamic Moment and Torque Diagrams on the Main Shaft with the Max Load Case

Table 7. Main Shaft Fatigue Analysis Values

Initial Configuration - Max Load Case (60RPM – 300lb Rider)

	CP1	**CP2**	**CP3**
Distance from z=0 [in]	39.04	35.5	31.25
Outer Width of Shaft [in]	5	5	5
Bending Stress Alternating [psi]	1864.6	2040.3	1829.4
Bending Stress Midrange [psi]	8993.2	9989.3	8793.4
Torsional Stress Alternating [psi]	3831.6	3831.6	3831.6
Torsional Stress Midrange [psi]	0.0	0.0	0.0
Von Mises Alternating [psi]	9997.4	6943.0	2793.3
Von Mises Midrange [psi]	8993.2	9989.3	8793.4
Static Safety Factor	2.6	2.9	1.4
Fatigue Safety Factor	1.6	2.0	0.6

The time response for the bending stresses at Critical Point 1 (the filleted location where the "bird beak" SHS T joint is attached) for maximum load case and the time response for the torsional stresses (shear) at Critical Point 1 can be seen in Figure 19.

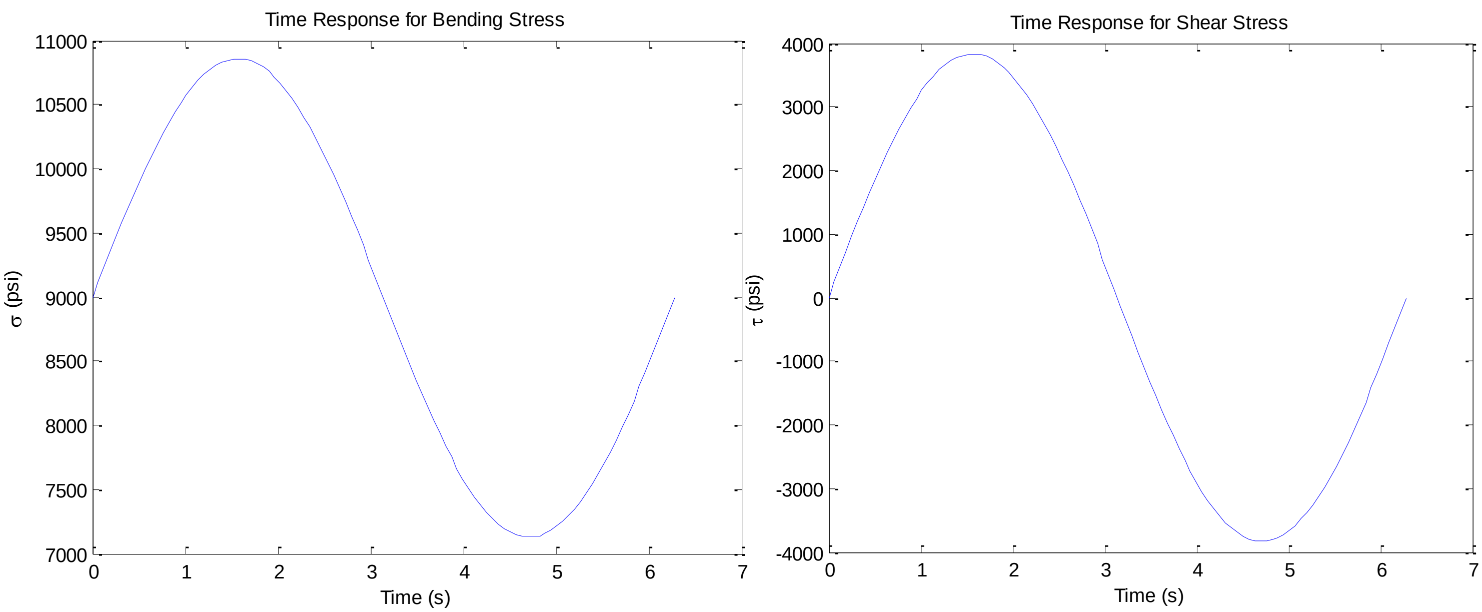


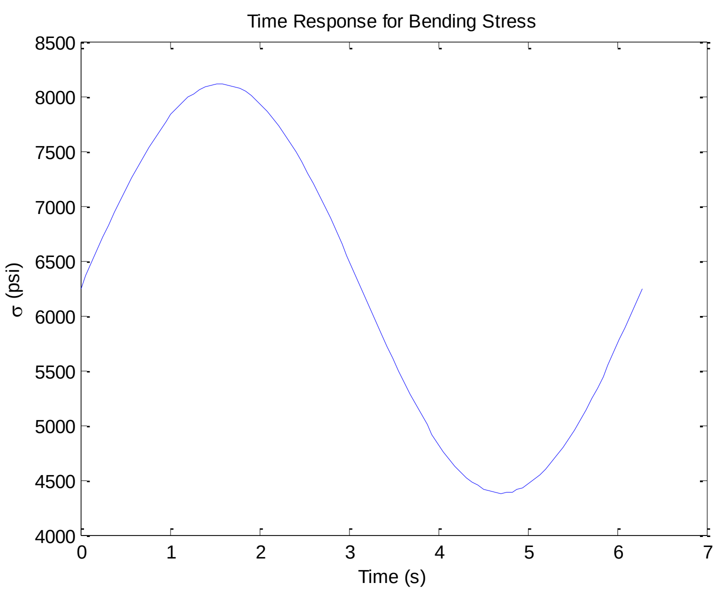
Figure 19 – Time Response Curves for Bending Stress (Left) and Shear Stress (Right) for the Max Load Case

Due to the assumption that 50RPM is the maximum non-lethal force we re-evaluated the critical points and received the following results.

Table 8. Main Shaft Fatigue Analysis Values

Initial Configuration - Non-Lethal Load Case (50RPM – 300lb Rider)

	CP1	**CP2**	**CP3**
Distance from z=0 [in]	39.04	35.5	31.25
Outer Width of Shaft [in]	5	5	5
Bending Stress Alternating [psi]	1864.6	2040.3	1829.4
Bending Stress Midrange [psi]	6245.3	6937.0	6106.5
Torsional Stress Alternating [psi]	3831.6	3831.6	3831.6
Torsional Stress Midrange [psi]	0.0	0.0	0.0
Von Mises Alternating [psi]	9997.4	6943.0	2793.3
Von Mises Midrange [psi]	6245.3	6937.0	6106.5
Static Safety Factor	3.1	3.6	1.5
Fatigue Safety Factor	1.7	2.2	0.7



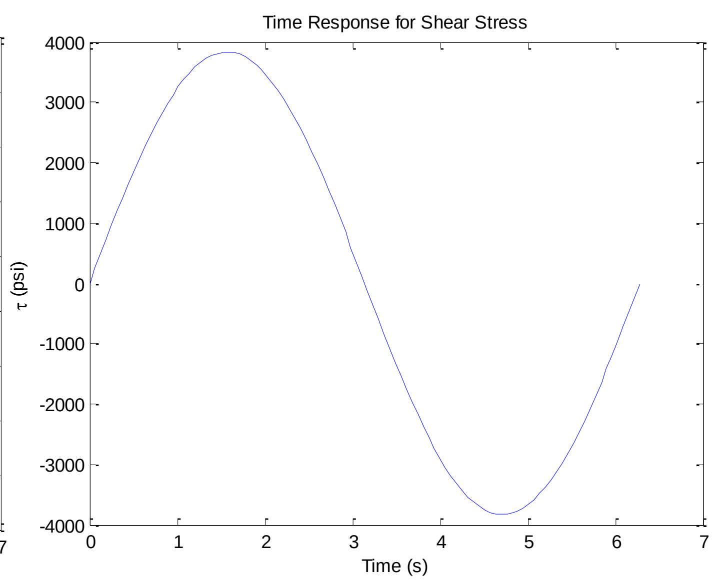


Figure 20 - Time Response Curves for Bending Stress (Left) and Shear Stress (Right) for the Non- Lethal Load Case

B. Pin

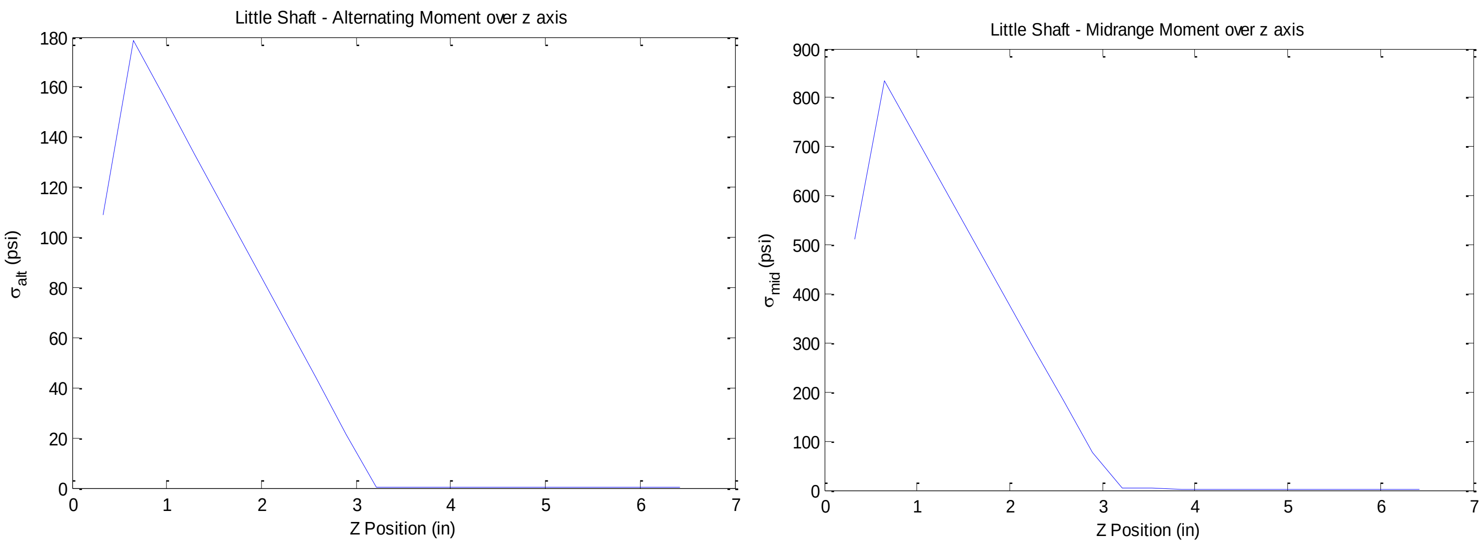


Figure 21 - Dynamic Moment Diagrams for the Pin with the Max Load Case

Table 9. Pin Fatigue Analysis Values

Initial Configuration - Max Load Case (60RPM – 300lb Rider)

	CP4	**CP5**	**CP6**
Distance from z=0 [in]	1.34	0.54	3.55
Diameter of Shaft [in]	1.99	2.01	1.75
Bending Stress Alternating [psi]	316.4	435.6	~0
Bending Stress Midrange [psi]	1459.9	2036.9	~0
Von Mises Alternating [psi]	316.4	435.6	~0
Von Mises Midrange [psi]	1459.9	2036.9	~0
Static Safety Factor	33.9	24.3	~∞
Fatigue Safety Factor	31.7	22.9	~∞

The maximum bending moment on the pin (labled little shaft in the diagram) can be clearly seen in the two previous figures. It lies immediately before the inner bearing and corresponds to Critical Point 5.Figure 22 shows the time response of the bending stess at Critical Point 5. In fact, this point has the lowest safety factor on the pin and it is sigfnicantly above 1. Even with stress concentration factors further on the pin the safety factors tend to infinity as the moments are so small. Due to the assumption that 50RPM is the maximum non-lethal force we re-evaluated the critical points and received the following results.

Table 10. Pin Fatigue Analysis Values

Initial Configuration - Non-Lethal Load Case (50RPM – 300lb Rider)

	CP4	CP5	CP6
Distance from z=0 [in]	1.34	0.54	3.55
Diameter of Shaft [in]	1.99	2.01	1.75
Bending Stress Alternating [psi]	316.4	435.6	~0
Bending Stress Midrange [psi]	1013.8	1414.5	~0
Von Mises Alternating [psi]	316.4	435.6	~0
Von Mises Midrange [psi]	1013.8	1414.5	~0
Static Safety Factor	45.3	32.6	~∞
Fatigue Safety Factor	38.8	27.9	~∞

As expected, these safety factors are even higher. This pin is therefore a target for re-design and weight reduction.

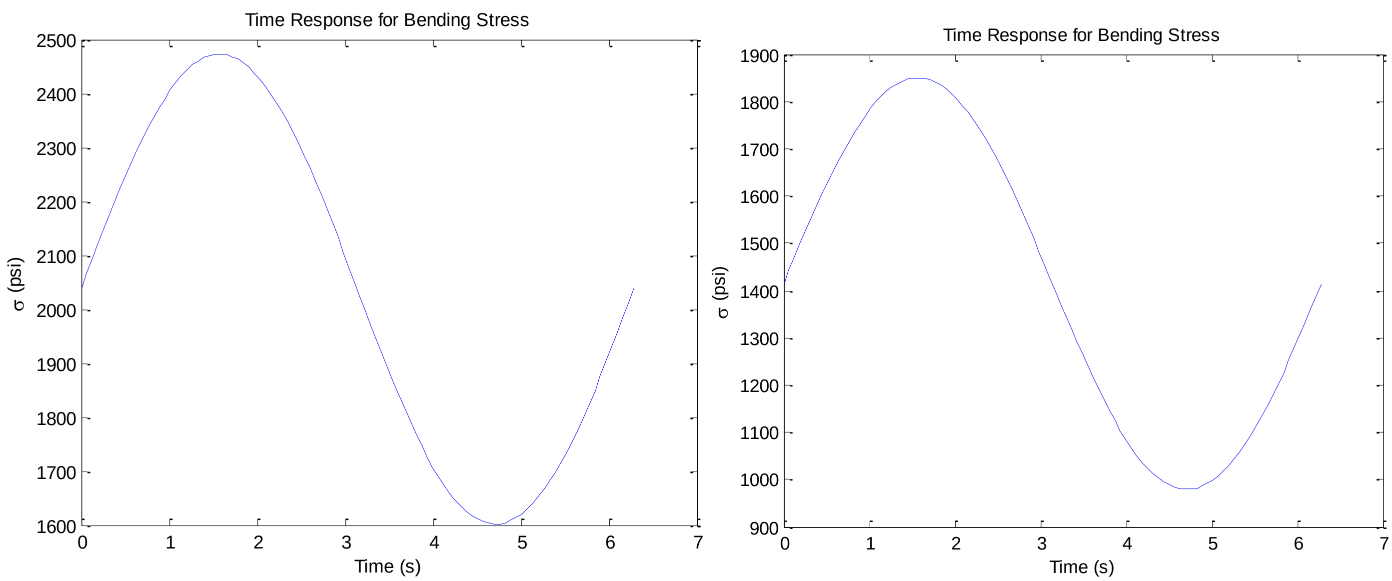


Figure 22 - Time Response Curves for Bending Stress for the Max Load Case (Left) and for Bending Stress for the Non- Lethal Load Case (Right)

C. Secondary Shaft

The fatigue analysis results were run for various angular speeds and weights. They demonstrate how the forces, moments and their corresponding stresses affect the yield and fatigue safety factors. The initial design fails with a fatigue safety factor of 0.7387 and 0.7325 at the press fit and bearing A, respectively. Lowering the RPM to 50 RPM affected the magnitude of the centripetal force and lowered the axial stress by 200 psi and provided insufficient reduction for the safety factor.

Table 11. Secondary Shaft Fatigue Analysis

Initial Configuration - Max Load Case (60RPM – 300lb Rider)

	At press fit	**Bearing A**	**Bearing B**	**Bolt**
Distance from top [in]	0	2.5600	5.1300	5.5000
Cross Sectional Area [in^2]	3.1416	3.1416	3.1416	3.1416
Moment [lbf in]	-20817.0	-19985.0	0	~0
Axial Force [lbf]	-1801.6	-1801.6	-1801.6	-1801.6
Radial Force [lbf]	325.3	7451.0	-7776.1	0
Stress due to moment [psi]	-26505.0	-25445.0	0	~0
Stress due to axial force [psi]	-573.5	-573.5	-573.5	-573.5
Von Mises Alternating [psi]	26627.0	2684.0	548.1	7448.4
Von Mises Midrange [psi]	677.0	710.9	3046.5	677.0
Fatigue Safety Factor	**0.7387**	**0.7325**	**13.7311**	**2.5914**

Table 12. Secondary Shaft Fatigue Analysis

Initial Configuration - Non-Lethal Load Case (50RPM – 300lb Rider)

	At press fit	**Bearing A**	**Bearing B**	**Bolt**
Distance from top [in]	0	2.5600	5.1300	5.5000
Cross Sectional Area [in^2]	3.1416	3.1416	3.1416	3.1416
Moment [lbf in]	-20817.0	-19985.0	0	~0
Axial Force [lbf]	-1151.7	-1151.7	-1151.7	-1151.7
Radial Force [lbf]	325.3	7451.0	-7776.1	0
Stress due to moment [psi]	-26505.0	-25445.0	0	~0
Stress due to axial force [psi]	-366.6	-366.6	-366.6	-366.6
Von Mises Alternating [psi]	26627.0	26845.0	548.1	7448.4
Von Mises Midrange [psi]	470.1	493.7	2115.7	470.1
Fatigue Safety Factor	**0.7404**	**0.7342**	**16.9417**	**2.6121**

D. Bearings

All six (6) bearings used for this swing are identified earlier in the bearing section and have the characteristics found in Table 13. The application factor is assumed to be 1 for all of the results. The life of the bearing is assumed to be 10^6, while the C_{90} is calculated for a life of $90x10^6$ therefore X_D= 1/90. Bearing A is the bearing that takes both the induced axial load from bearing Band the external axial load. Bearing B takes no axial load.

Table 13. Bearing Specifications

Reliability	0.995
a	10/3
b	1.483
θ	4.459
X_0	0.02
Application factor	1
C_{90} (radial) [lbs]	5610

1. Pin Bearings

One of the assumptions identified earlier was that of no or very little deflections and slopes. This is important because bearings have a maximum slope allowed. In the case of the deep groove ball bearing the maximum slopes are 0.000372 rad at 60RPM and 0.000278 rad at 50 RPM.

This assumption was approximately verified by modeling a simply supported beam on either side of the inner bearing; the location of the maximum moment. A slope equation for this model was coded into Matlab and the simulation output the following results.

Table 14. Pin Bearing Results
Initial Configuration - Max Load Case (60RPM – 300lb Rider)

	Inner Bearing (A)	**Outer Bearing (B)**
Magnitude of Radial Force [lbf]	2335.8	399.03
Magnitude of External Axial Force [lbf]	0	0
Induced Axial Load [lbf]	98.19	0
Effective (Radial) Load [lbf]	1121.9	399.03
Max Catalogue Radial Load [lbf]	11766	11766
Safety Factor	10.49	29.49

Table 15. Pin Bearing Results
Initial Configuration - Non-Lethal Load Case (50RPM – 300lb Rider)

	Inner Bearing (A)	**Outer Bearing (B)**
Magnitude of Radial Force [lbf]	1747.3	296.07
Magnitude of External Axial Force [lbf]	0	0
Induced Axial Load [lbf]	72.85	0
Effective (Radial) Load [lbf]	838.07	296.07
Max Catalogue Radial Load [lbf]	11766	11766
Safety Factor	14.04	39.75

Note that the safety factor on the outer bearing is significantly larger than that of the inner bearing. This is due to the lower reaction forces due to the lower moment at that location. The inner bearing is therefore the critical bearing, and if both bearings are to be identical then there is significant maneuverability for bearing selection, given the same forces. The bearings do not fail at maximum loads. Since there is no precise safety factor for bearings, the safety factors indicated in these table is a ratio of the maximum radial force allowable that achieves a life of 10^6cycles to the equivalent radial force on the bearing due to applied forces and the geometry. The maximum radial force allowable that achieves a life of 10^6cycles is calculated using C_{90} given in the catalogue and the Weibull values in the table above.

2. *Secondary Shaft Bearings*

Table 16. Secondary Shaft Bearing Results
Initial Configuration - Max Load Case (60RPM – 300lb Rider)

	Inner Bearing (B)	Outer Bearing (A)
Magnitude of Radial Force [lbf]	7451.8	7776.1
Magnitude of External Axial Force [lbf]	1801.6	1801.6
Induced Axial Load [lbf]	0	1833.7
Effective (Radial) Load [lbf]	7451.8	10054
Max Catalogue Radial Load [lbf]	11766	11766
Safety Factor	1.58	1.17

Table 17. Secondary Shaft Bearing Results
Initial Configuration - Non-Lethal Load Case (50RPM – 300lb Rider)

	Inner Bearing (B)	Outer Bearing (A)
Magnitude of Radial Force [lbf]	7451.8	7776.1
Magnitude of External Axial Force [lbf]	1151.7	1151.7
Induced Axial Load [lbf]	0	1833.7
Effective (Radial) Load [lbf]	7451.8	8812.5
Max Catalogue Radial Load [lbf]	11766	11766
Safety Factor	1.58	1.34

The safety factors on the bearings of the secondary shaft are much smaller those on the pin however these bearings still do not fail. Therfore, a re-design is not necessary unless the shaft itself is re-designed. The latter happens to be the case therefore new bearings will be selected for both the secondary shaft and the pin. Note that on the secondary shaft the outer bearing is assumed to take the external axial load and the induced axial load from the radial load on in the inner bearing.

E. Bolted Joints

1. BJ-6

Using table 8-10 Shigley's Mechanical Engineering Design 9th Edition.

Table 18. Material Properties

Bolt Yield Strength (ksi)	95.72
Bolt Static Yield Strength (ksi)	55.23

Table 19.

	50 RPM	**60 RPM**
Shear Area (in^2)	0.7854	0.7854
Shear Stress (ksi)	359.638	553.93
Safety Factor on Shear of Bolts	**15.357**	**9.9705**

Table 20.

	50 RPM	**60 RPM**
Bearing Area on Bolts (in^2)	1	1
Bearing stress (ksi)	-282.44	-435.035
Safety Factor of bearing on bolts	**33.891**	**22.003**

Table 21.

	50 RPM	**60 RPM**
Member Yield Strength (ksi)	60.19	60.19
Safety factor on members	**21.310**	**13.836**

Table 22.

	50 RPM	**60 RPM**
Tensile area (in^2)	0.615	0.615
Tensile Stress (ksi)	459.249	707.356
Safety Factor from tension	**13.101**	**8.506**

2. BJ-2

Using table 8-10 Shigley's Mechanical Engineering Design 9th Edition.

Table 23. Material Properties

Bolt Yield Strength (ksi)	95.72
Bolt Static Yield Strength (ksi)	55.23

Table 24.

	50 RPM	**60 RPM**
Shear Area (in^2)	1.5708	1.5708
Shear Stress (ksi)	574.76	1508.149
Safety Factor on Shear of Bolts	**19.218**	**7.324**

Table 25.

	50 RPM	**60 RPM**
Bearing Area on Bolts (in^2)	2	2
Bearing stress (ksi)	-451.42	-1184.5
Safety Factor of bearing on bolts	**21.204**	**8.081**

Table 26.

	50 RPM	60 RPM
Member Yield Strength (ksi)	42.5	42.5
Safety factor on members	**14.399**	**5.488**

Table 27.

	50 RPM	60 RPM
Tensile area (in^2)	1.854	1.854
Tensile Stress (ksi)	486.965	1277.78
Safety Factor from tension	**13.860**	**5.087**

The results above make sense since there are 3 fasteners taking on the load from the main shaft and they are all 1 inch in diameter or more. Furthermore, at extreme load cases it can be seen that the safety factor decreses and this is also due to the fact that the material of the member attached to the fastners is aluminum. Aluminum has a lower yield strength as compared to steel.

Table 28.

Factor of Safety against bolt failure	**12.26**
Factor of Safety against bolt separation	**18.82**
Fatigue factor of safety using Goodman criteria	**2.475**
Fatigue factor of safety using Gerber criteria	**38.386**
Fatigue factor of safety using ASME-elliptic criteria	**3.189**

3. BJ-3

Material Properties:
Using table 8-10 Shigley's Mechanical Engineering Design 9th Edition.

Table 29. Material Properties

Bolt Yield Strength (ksi)	95.72
Bolt Static Yield Strength (ksi)	55.23

Table 30.

	50 RPM	60 RPM
Shear Area (in^2)	0.7854	0.7854
Shear Stress (ksi)	-1147.65	-1795.26
Safety Factor on Shear of Bolts	**19.248**	**12.305**

Table 31.

	50 RPM	60 RPM
Bearing Area on Bolts (in^2)	0.87	0.87
Bearing stress (ksi)	1036.05	1620.69
Safety Factor of bearing on bolts	**14.035**	**11.45**

Table 32.

	50 RPM	60 RPM
Member Yield Strength (ksi)	65	65
Safety factor on members	**6.27**	**4.01**

Table 33.

	50 RPM	60 RPM
Tensile area (in^2)	0.76	0.76
Tensile Stress (ksi)	1186	1855.26
Safety Factor from tension	**5.48**	**3.504**

4. *BJ-5*

Table 33b.

Factor of Safety against bolt failure	**12.26**
Factor of Safety against bolt separation	**18.82**
Fatigue factor of safety using Goodman criteria	**2.475**
Fatigue factor of safety using Gerber criteria	**38.386**
Fatigue factor of safety using ASME-elliptic criteria	**3.189**

These results seem to be very credible since there are 4 fasteners that are taking the same shear load. Furthermore, there is a lot of shear force applied to these 4 fasteners due to the secondary shaft, hence, the bolts are not as over designed as compared to other ones.

VII. Re-Design

The re-design process involved ensuring that the safety factors for our design were reliably around 1- 1.5. The main changes required was to reduce weight. Due to the large load case it was not possible to remove much weight off of the swing. The initial weight of the swing was 696.943lbf. We were able to reduce the weight to 672.62lbf this was a weight savings of 3%. The re-design of each specific components is detailed in the following paragraphs.

A. Main Shaft

The main shaft was one of the largest re-designs carried out. It was decided to first model a circular tube instead of the square SHS configuration (concept can be seen in Figure 23). However it was not possible to reduce the weight in the configuration without receiving a failing safety factor with the cylindrical cinfiguration. For this reason we decided to return to the initial square shaft configuration and focus on minimizing the volume of the shaft as well as removing any stress risers. After attempting multiple wall thicknesses and widths it was found that the most weight could be lost, without sacrificing any safety, by decreasing the width by 0.5" and keeping the wall thickness to the initial thickness of 0.19". Additionally all corners on the shaft were filleted with a radii of 0.40" and the inside radii was increased from 0.20" to 0.40" to eliminate stress inside of the shaft. The largest stress riser in the main shaft was the large hole through the bottom of the shaft to allow for an emergency brake. The proposed redesigned shaft no longer has a hole and instead the braking system for the VIROMAX will be built onto the shaft externally to avoid creating a large stress riser. For this reason Table 34 and 35 no longer evaluate the forces at CP3. A conceptual drawing can be seen in Figure 24 and the full dimensions can be seen in Appendix C. Since the width of the shaft was decreased we also were required to decrease the size of the square attachments that fit into the shaft These were drafted in Solidworks specifically to determine their new volumes, this resulted in a los of approximately 2lb per attachment and therefore 4lb of additional weight decrease. In the end the new configuration allowed us to decrease the weight of the primary shaft (square attachments and fasteners included) from 95.4lbf to 80.07lbf which was approximately a 16% weight loss.

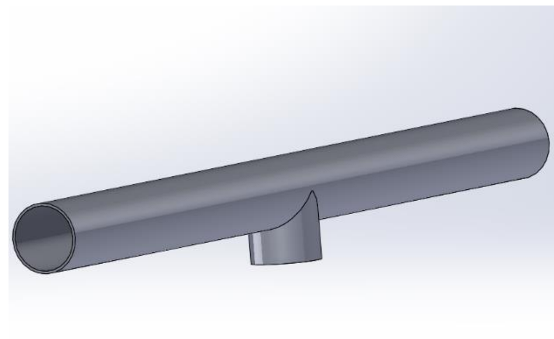


Figure 23 – CircularMain Shaft Conceptual Drawing

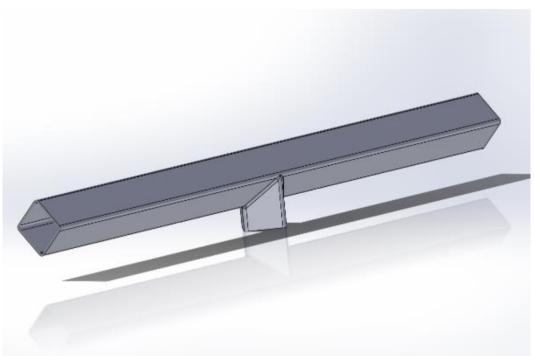


Figure 24 – Final Redesign – Outer Width 4.5"

Table 34. Main Shaft Fatigue Analysis Values

Re-Design Outer Width=4.5"- Max Load Case (60RPM – 300lb Rider)

	CP1	**CP2**
Distance from z=0 [in]	39.04	35.5
Outer Width of Shaft [in]	4.5	4.5
Bending Stress Alternating [psi]	2293.0	2508.5
Bending Stress Midrange [psi]	11360.0	12618.0
Torsional Stress Alternating [psi]	4866.7	4866.7
Torsional Stress Midrange [psi]	0.0	0.0
Von Mises Alternating [psi]	12685.0	8794.7
Von Mises Midrange [psi]	11360.0	12618.0
Safety Factor Yield	2.1	2.3
Safety Factor Fatigue	1.2	1.6

Table 35. Main Shaft Fatigue Analysis Values

Re-Design Outer Width=4.5" - Non-Lethal Load Case (50RPM – 300lb Rider)

	CP1	**CP2**
Distance from z=0 [in]	39.04	35.5
Outer Width of Shaft [in]	4.5	4.5
Bending Stress Alternating [psi]	2293.0	2508.5
Bending Stress Midrange [psi]	7888.6	8762.4
Torsional Stress Alternating [psi]	4866.7	4866.7
Torsional Stress Midrange [psi]	0.0	0.0
Von Mises Alternating [psi]	12685.0	8794.7
Von Mises Midrange [psi]	7888.6	8762.4
Static Safety Factor	2.4	2.8
Fatigue Safety Factor	1.3	1.7

B. Pin

Since the Pin safety factors are much higher than 1, weight can be saved by redesigning the pin. To facilitate this process the Matlab Slopes code was used to determine the minimum wall thickness of the pin. This was done in order to guarantee a maximum slope of 0.0005 rad (a requirement for use of tapered roller bearings). The maximum interior diameter, keeping the outer diameter constant was found to be 1.4 inches. Because only the wall thickness of the pin was changed (along with elimination of unnecessary diameter drops) the new pin can be used in conjunction with all of the same parts such as the press fit square sleeve and the parts connecting the pin to the legs. A weight savings of 45% of the original pin is achieved in this way. Note that the notch preventing the threaded nut from contacting the bearings is kept in the same location. This notch is not in a critical position because of the severely reduced moment after the inner bearing.

Table 36. Pin Fatigue Analysis Values

Re-Design Remove 1.99"Step Change - Max Load Case (60RPM – 300lb Rider)

	CP4	CP5	CP6
Distance from z=0 [in]	1.34	0.54	3.55
Diameter of Shaft [in]	2.00	2.01	1.75
Bending Stress Alternating [psi]	6015.2	8177.9	~0
Bending Stress Midrange [psi]	28492.0	39257.0	~0
Von Mises Alternating [psi]	6015.2	8177.9	~0
Von Mises Midrange [psi]	28492.0	39257.0	~0
Static Safety Factor	1.7	1.3	~∞
Fatigue Safety Factor	1.6	1.2	~∞

Due to the assumption that 50RPM is the maximum non-lethal force we re-evaluated the critical points and received the following results.

Table 37. Pin Fatigue Analysis Values

Re-Design Remove 1.99"Step Change - Non-Lethal Load Case (50RPM – 300lb Rider)

	CP4	CP5	CP6
Distance from z=0 [in]	1.34	0.54	3.55
Diameter of Shaft [in]	2.00	2.01	1.75
Bending Stress Alternating [psi]	6015.2	8177.9	~0
Bending Stress Midrange [psi]	19786.0	27262.0	~0
Von Mises Alternating [psi]	6015.2	8177.9	~0
Von Mises Midrange [psi]	19786.0	27262.0	~0
Static Safety Factor	2.3	1.7	~∞
Fatigue Safety Factor	2.0	1.5	~∞

Note that the fatigue and static safety factors on the pin are at much lower but still satisfactory levels given that these are extreme loading cases.

C. Secondary Shaft

First, for improving static safety factor, the material of the secondary shaft could be altered to ensure it doesn't yield. Unfortunately, for increasing Sy and Sut values the fatigue safety factor did not increase considerably.

Table 38. Comparison of Fatigue Safety Factors at Bearing A for Various Materials
Initial Configuration - Max Load Case (60RPM – 300lb Rider)

Material	Sy (ksi)	Sut (ksi)	Safety Factor
Steel	33.4	67.45	0.7325
Carbon Steel	78.3	60.2	0.6736
Mild Carbon Steel	120.0	95.7	0.9479

While keeping the original material, the simplest and most convenient method of improving safety factor was by increasing the diameter of the shaft to 2.5 inches. The overall volume would increase from 27.75 inches cubed to 42.36 inches cubed, resulting in an increase of mass of 4.15 lbs.

Table 39. Secondary Shaft Fatigue Analysis
Redesigned at d=2.5"- Max Load Case (60RPM – 300lb Rider)

	At press fit	Bearing A	Bearing B	Bolt
Distance from top [in]	0	2.5600	5.1300	5.5000
Cross Sectional Area [in^2]	4.9087	4.9087	4.9087	4.9087
Moment [lbf in]	-2.0817e+04	-1.9985e+04	0	-3.6380e-12
Axial Force [lbf]	-1.8016e+03	-1.8016e+03	-1.8016e+03	-1.8016e+03
Radial Force [lbf]	325.3	7.4508e+03	-6,085.6	0
Stress due to moment [psi]	-1.3571e+04	-1.3028e+04	7.7761e+03	-2.3716e-12
Stress due to axial force [psi]	-367.0236	-367.0236	-367.0236	-367.0236
Von Mises Alternating [psi]	1.3649e+04	1.3761e+04	350.8056	3.8292e+03
Von Mises Midrange [psi]	433.2869	454.9512	1.9498e+03	433.2869
Static Safety Factor	0.0024	0.0023	0.0145	0.0078
Fatigue Safety Factor	1.4385	1.4262	21.4549	5.0088

Table 40. Comparison of Yield and Fatigue Safety Factors for two diameters
Max Load Case (60RPM – 300lb Rider)

Diameter	Safety Factor	At press fit	Bearing A	Bearing B	Bolt
2.0	Yield	1.2233	1.2121	9.2915	4.1106
	Fatigue	0.7387	0.7325	13.7311	2.5914
2.5	Yield	2.3718	2.3495	14.5180	7.8358
	Fatigue	1.4385	1.4262	21.4549	5.0088

D. Bearings

New tapered roller bearings were chosen from the same Timken catalogue for the secondary shaft. New ones had to be chosen even though the originals did not fail for the same loads. This is because the new design for the secondary shaft prescribes a 2.5 inch bore diameter as opposed to 2 inches. A 2.5 inch bore diameter was therefore the first requirement. The Matlab code was then used to determine the appropriate K value (ratio of radial load rating to axial load rating, as defined by Timken) and the $C_{90.}$ The smallest bearing meeting these requirements was chosen. To simplify the changing of the bearings for maintenance, and the procurement of the bearings, identical bearings were chosen. The calculations were done for a bearing achieving 10^6 cycles (the C_{90} rating was converted to 10^6, i.e XD = 1/90). This gives equivalent results to using a C_{10} rating with XD=1 .

Bearing Dimensions			Load Ratings							Part Number	
Bore d	O.D. D	Width T	Dynamic(1) C_1	Factors(2) e	Y	Dynamic(3) C_{90}	C_{a90}	Factors(2) K	Static C_0	Inner	Outer
mm in.	mm in.	mm in.	N lbf			N lbf	N lbf		N lbf		
63.500 2.5000	**104.775** 4.1250	**21.433** 0.8438	**115000** 25800	0.39	1.55	**29700** 6680	**19700** 4440	1.51	**120000** 27000	39250	39412

Figure 25 -

1. Pin Bearings

Since the pin re-design maintains the same 2 inch bore diameter that was the first design requirement for the bearings. The C90 and K values were then iterated to see which bearing met all of the requirements. The smallest bearing for this bore diameter was chosen because it adequately supported the maximum loads. A marginal weight savings is therefore achieved as the 4 bearings needed are all smaller than the assumed original model. The calculations were done for a bearing achieving 10^6 cycles (the C_{90} rating was converted to 10^6, i.e XD = 1/90). This gives equivalent results to using a C_{10} rating with XD=1.

Bearing Dimensions			Load Ratings							Part Number	
Bore d	O.D. D	Width T	Dynamic(1) C_1	Factors(2) e	Y	Dynamic(3) C_{90}	C_{a90}	Factors(2) K	Static C_0	Inner	Outer
mm in.	mm in.	mm in.	N lbf			N lbf	N lbf		N lbf		
50.800 2.0000	**77.788** 3.0625	**12.700** 0.5000	**37300** 8390	0.34	1.78	**9680** 2180	**5570** 1250	1.74	**47200** 10600	LL205449	LL205410

Figure 26 -

2. Slopes on the bearings:

<u>Secondary Shaft:</u>

The maximum slope calculated for the new secondary shaft is $2.57\text{x}10^{-4}$ rad at any point on the shaft. This slope is safely under the maximum slope recommended for a tapered roller bearing (0.0005 rad according to Shigley).

<u>Pin:</u>

The maximum slope calculated for the pin is $4.9\text{x}10^{-4}$ rad. This also is safely under 0.0005 rad and therefore, tapered roller bearings can be safely used on the pin. Note that this was already a known as the maximum slope was used as a requirement in the pin redesign process.

E. Bolted Joints

Intuitively, it was known that the bolts were overdesigned. This was a safe assumption that was later confirmed when analyzing the safety factors based on engineering calculations. Furthermore, the majority of these fasteners had a diameter of 1 inch which is really big!
After redesigning, these are the results that are obtained:

Table 41.

	BJ-6		**BJ-3**		**BJ-2**	
	50 RPM	**60 RPM**	**50 RPM**	**60 RPM**	**50 RPM**	**60 RPM**
Safety factor from shear	2.79	1.813	3.215	2.113	4.154	3.762
Safety factor of bearing on bolts	6.162	4.00	2.734	1.977	5.346	4.555
Safety factor on members	3.84	2.5	1.816	1.773	2.991	2.623
Safety factor from tension	2.302	1.55	1.826	1.237	2.092	2.006

This particular bolt was found on McMaster as a standard component and was used to replace the bolts on BJ-6 and BJ-2. Some information found on their website states that this bolt is "An economical alternative to high-strength steel cap screws, these screws are at least 60% as strong. Inch screws are marked on the head with three radial lines to indicate Grade 5 and have a Class 2A thread fit. Minimum tensile strength is 120,000 psi and minimum Rockwell hardness is C25. Screws meet ASME B18.2.1 and SAE J429." The reason for choosing grade 5 bolts is because it was not necessary for the bolts to have a strength matching with grade 8.8 specifications. Furthermore, these bolts are also weather and corrosion resistant which makes it ideal for all seasons.

The redesigned fastner for BJ-3 can be seen in Appendix C.

In the redesign, it was decided to reduce the diameter of the bolts since their safety factor was too high and because it is more economical to use standard off the shelf fasteners. Both new fasteners were basically identical in terms of material and properties, however their diameter was different. By reducing the diameter, we were able to save weight on the overall structure as well as reduce the cost of components. Lastly, the redesigned fasteners were Off the Shelf Components which means they are more accessible and they are cheaper to use since they are standard parts.

VIII. Conclusion

After a full static and dynamic analysis of the VIROMAX it was concluded that the swing was in fact over designed in terms of the main shaft and pin as could be seen from the large safety factors (other than the CP3 safety factor however it was decided that this was due to an extremely over conservative stress concentration factor). However the secondary shaft did fail at the maximum loads (i.e. with 60 RPM and a 300lb rider) with not very conservative concentration factor. Upon further investigation it was found that a human body is not capable of sustaining a the G force for a steady state condition. Therefore we decided to amend the loads and design for a realistic load case, even though the 50 RPM could not be sustained for more than 2 seconds by an untrained human. These large forces required the secondary shaft to have an increased diameter such that it would not fail in infinite life resulting in an increase of 4.5lbs. It was possible to decrease the weight of the pins and the main shaft though and in total the weight savings were 23lbs which was approximately a 3% decrease in weight.

.

IX. Appendix

A. MATLAB Codes

1. Bearing_tapered

```
function [ inner_ratio, outer_ratio ] = bearing_tapered(F_axial_external, F_radial_inner,
F_radial_outer,pin_secondary)
% inputs : axial force, radial force on inner and outer bearings, designate
% if pin (1) or secondary shaft (2)
%

Rd= 0.995;
Xd=1;

Xo=0;
theta=4.48;
b=1.5;
a=10/3;

% from catalogues
C_90= 2180;
KA=1.74;
%C_90=6680;
%KA=1.51;
%KB=1.51;
KB=1.74;


a_f=1;
F_max = (C_90/a_f)*((Xo + (theta-Xo)*log(1/Rd)^(1/b))/(Xd))^(1/a)



if pin_secondary == 1 % i.e pin: inner = A, outer = B


    FrA = F_radial_inner;
    FrB = F_radial_outer;

    FeA= 0.4*FrA + KA*((0.47*FrB/KB) + F_axial_external)
    Fin=(0.47*FrB/KB)
    FeB=FrB

    inner_ratio = F_max/FeA;
    outer_ratio = F_max/FeB;

else % i.e secondary shaft: inner = B, outer = A

    FrB = F_radial_inner;
    FrA = F_radial_outer;

    FeA= 0.4*FrA + KA*((0.47*FrB/KB) + F_axial_external)
    Fin=(0.47*FrB/KB)
    FeB=FrB

    inner_ratio = F_max/FeB;
    outer_ratio = F_max/FeA;

end

end
```

2. Slope_cantilevered_and_simple_moment

```
function [max_slope] = slope_cantilevered_and_simple_moment(F,d_o,d_i)
%inputs: F = REACTION FORCE FROM INNER BEARING
%        x = point where slope needs to be calculated
%outputs: axial stress, bending stress and torsional shear stress

I=(pi/64)*(d_o^4 -d_i^4);
```

```
E=29000000; %psi
L=3.115;
b=0.545;
a=L-b;
z_bi=0.545;
M= z_bi*F;

for i=1:1:31;
    x=i/10;
slope(i) = abs(((2*F*x)/(6*E*I))*(x -3*L) + ((F*x^2)/(6*E*I)) + (M/(6*E*I*L))*(x^2 + 3*a^2 -6*a*L
+2*L^2) + (M*x*(2*x)/(6*E*I*L)));
end
max_slope = max(slope);
end
```

3. Slope_cantilevered

```
function [max_slope] = slope_cantilevered(Fi,Fo,d_o,d_i)
%inputs: F = REACTION FORCE FROM INNER BEARING
%        x = point where slope needs to be calculated
%outputs: axial stress, bending stress and torsional shear stress

I=(pi/64)*(d_o^4 -d_i^4);
E=29000000; %psi
L=3.115;
b=0.545;
a=L-b;

for i=1:1:51;
    x=i/10;
slope_AB(i) = ((2*Fi*x)/(6*E*I))*(x -3*a) + ((Fi*x^2)/(6*E*I)) +((2*Fo*x)/(6*E*I))*(x-3*L)+
((Fo*x^2)/(6*E*I)) ;
slope_BC(i) = ((Fi*a^2)/(6*E*I))*(-3) + ((Fo*x^2)/(6*E*I)) + ((2*Fo*x)/(6*E*I))*(x-3*L);

if x < b
    slope(i) = slope_AB(i);
else
    slope(i) = slope_BC(i);
end

end
slope = abs(slope);
max_slope = max(slope);

end
```

4. Calculate_Se

```
function [Se] = calculate_Se(Sut,load_type, surface_cond, cross_section_shape, d,o_w,i_w)
%inputs:  Sultimate, load type, surface condition, cross section shape of
%shaft,diameter of circular shaft, outer width, inner width
%load_type =
%1 for bending
%2 for torsion
%3 for axial
%surface_cond =
%1 for machine/CD
%2 for ground
%3 for as-forged
%4 for Hot Rolled
%cross_section_shape
%1 circle
%2 hollow square
%outer width and inner width
%if shape is circular then enter anything (1 and 1)
%if shape is hollow square then enter correct values
%function calculates Se_p = Se_prime, ka, kb, kc, kd, ke, kf  to get Se
%outputs: Se for infinite life
% Se_p calculation
if Sut< 200
    Se_p=0.5*Sut;
```

```
else
    Se_p=100;
end

%ka: suface factor
if surface_cond==2%i.e ground
    a= 1.34;
    b=-0.085;
end

if surface_cond==3 %i.e as-forged
    a= 39.9;
    b=-0.995;
end

if surface_cond==4 %i.e HR
    a= 14.4;
    b=-0.718;
else               %i.e machine/CD or default
    a= 2.70;
    b=-0.265;
end
ka=a*Sut^b;

%kb: size factor
 if cross_section_shape == 2 % hollow square, getting equivalent diameter
    de = 0.808*(o_w^2 - i_w^2)^(0.5);
    d=de;
 else
     d=0.370*d;
 end

 if d < 2 % if diameter is less than 2 inches
    kb=0.879*d^(-0.107);
 else   % diameter greater than 2 inches
    kb= 0.91*d^(-0.157);
 end

 if load_type == 3 % if only in axial
    kb=1;
 end


%kc: loading factor
 if load_type == 3 % axial
     kc=1;
 end

 if load_type ==2 % torsion
     kc=0.59;
 else             % bending/default
     kc = 1;
 end

 %kd: temperature factor note all shafts are made of steel
 %odd... most steels experience higher ultimate strength at low
 %temperatures, but they embrittle. do not know how to handle. at +40
 %celsius k = approx 1
kd=1;

%ke:Reliability Factor
%for 99% reliablity ke=0.814 check slide to change
ke=0.814;

%kf: miscellaneous
% ignoring corrosion, cyclic frequency affects at high temperatures etc.
kf=1;
Se=ka*kb*kc*kd*ke*kf*Se_p;

End
```

5. Fatigue_safety_factor_mod_goodman

```
function [sf_fatigue] = fatigue_safety_factor_mod_goodman(S_e,S_ut,sig_a_p,sig_m_p)
%inputs: S_e, S_ut andvon mises sig_a_prime, sig_m_prime
%outputs: safety factor for fatigue, infinite life

one_over_n= (sig_a_p/S_e) + (sig_m_p/S_ut);
sf_fatigue = (1/one_over_n);

end
```

6. FBD1_Sec_Shaft

```
function [At,Bt,Fbolt] = FBD1_Sec_Shaft(Rn,Rt,Mr,l1,l2)
%SEC_SHAFT Summary of this function goes here

%%Forces
Fbolt=Rn;
At=(Mr-Rt*(l2))/(l2-l1);
Bt=-Rt-At;
%Bx=(w^2*(64/12)*(400/32.2)+W)-Rn;

end
```

7. Little_shaft_dynamics

```
function [ Ma, Mm, Rboy, Rbiy, Rbox, Rbix, R_ob_mag, R_ib_mag ] = little_shaft_dynamics(Wls, Wn,
Ray, Rax, z_bo , z_bi, z_n, L, z)
%   inputs: weights, reaction force at A,  moment arms, shaft length,and z
%   z =the point you where want to calculate the moment. point C = (z=o, origin)
%   outputs: Ma, Mm, T_a, T_m, R_A_y (array), R_A_x (array), M (array) (no axial, R_A_z assumed
zero)
%


Rboy= zeros(360,1);
Rbiy= zeros(360,1);
Rbox= zeros(360,1);
Rbix= zeros(360,1);

for i = 1:1:360

    theta_d = i;
%     theta_r = theta_d*pi/180;

    % Bearing Reaction forces
    Rboy(i)= (1/(1-(z_bo/z_bi)))*(Ray(i)+ Wn*(1- z_n/z_bi) + Wls*(1-(L/2)/z_bi));
    Rbiy(i)= (- Rboy(i)*z_bo + Wls*(L/2) + Wn*z_n)/z_bi;
    Rbox(i)= Rax(i)/(1-(z_bo/z_bi));
    Rbix(i)= - Rbox(i)*z_bo/z_bi;

    % If statements based on z, for moment diagram
    if z < z_bi
        M_x = Ray(i)*z;
        M_y = -Rax(i)*z;
    else
        if z < z_bo
            M_x = Ray(i)*z - Rbiy(i)*(z-z_bi);
            M_y = -Rax(i)*z + Rbix(i)*(z-z_bi);
        else
            if z < L/2
                M_x = Ray(i)*z - Rbiy(i)*(z-z_bi) - Rboy(i)*(z-z_bo);
                M_y = -Rax(i)*z + Rbix(i)*(z-z_bi) + Rbox(i)*(z-z_bo);
            else
                if z < z_n
                    M_x = Ray(i)*z - Rbiy(i)*(z-z_bi) - Rboy(i)*(z-z_bo) + Wls*(z-(L/2));
                    M_y = -Rax(i)*z + Rbix(i)*(z-z_bi) + Rbox(i)*(z-z_bo);
                else
                    M_x = Ray(i)*z - Rbiy(i)*(z-z_bi) - Rboy(i)*(z-z_bo) + Wls*(z-(L/2)) + Wn*(z-
z_n);
                    M_y = -Rax(i)*z + Rbix(i)*(z-z_bi) + Rbox(i)*(z-z_bo);
```

```
                end
            end
        end
    end


    % Moment Magnitude
    M(i) = (M_x^2 +M_y^2)^(1/2);
    R_ob(i) = (Rbox(i)^2 +Rboy(i)^2)^(1/2);
    R_ib(i)=(Rbix(i)^2 +Rbiy(i)^2)^(1/2);
    % Torque
    %T(i) = (W_B*r_B - W_c*r_cw)*sin(theta_r);

end

Ma = abs((max(M) - min(M))/2);
Mm = (max(M) + min(M))/2 ;

R_ob_mag=max(R_ob);
R_ib_mag=max(R_ib);

%Ta = abs((max(T) - min(T))/2);
%Tm = (max(T) + min(T))/2 ;

end
```

8. Main_Script

```
%Main Script

%Pull Info from excel sheet
filename = 'MECH393.xlsx';
Sheet = 'MATLABInputs';
%Get Weights
WRange = 'B2:B8';
W = xlsread(filename,Sheet,WRange);
%Get Distances
DRange = 'E2:E28';
D = xlsread(filename,Sheet,DRange);

% F(1)=R_sb_y;
% F(2)=F_ob_s_y;
% F(3)=F_ib_s_y;
% F(4)=F_ob_y;
% F(5)=F_ib_y;
% F(6)=F_ot_y;
% F(7)=F_it_y;
% F(8)=Rx;
% F(9)=Ry;
% F(10)=Rz;
% F(11)=F_leg_x;
% F(12)=F_leg_y;
% F(13)=F_leg_z;

%%

% % % -----------------------STATICS----------------------------------------
% [ F ] = FBD_Solver( W(1), W(2), W(3), W(4), W(5), W(6), W(7), D(1), D(14),D(7),
D(3),D(4),D(5),D(6));
% for i=1:1:20
% z=.321*i;
% M_ls(i) = ls_moments( F(2),F(3),W(4),F(1),W(5),D(7),D(8),D(9),z,D(2));
% Z_ls(i)=z;
% end
%
% for i=1:1:20
% z=3.55*i;
% M_ms(i) = ms_moments(F(1), W(1), W(2),W(3),D(10),D(11),D(12),z);
% Z_ms(i)=z;
% end
```

```
% plot(Z_ls,M_ls);
% title('Little Shaft Static Moment over z axis')
% xlabel('z')
% ylabel('Moment (psi)')
% figure
%
% plot(Z_ms,M_ms);
% title('Main Shaft Static Moment over z axis')
% xlabel('z')
% ylabel('Moment (psi)')
%%

%-----------------------------------------------------------------------
%----------------------DYNAMICS---------------------------------------
%-----------------------------------------------------------------------
%
M_a_ms= zeros(1,20);
M_m_ms= zeros(1,20);
T_a_ms= zeros(1,20);
T_m_ms= zeros(1,20);
RA_y_ms= zeros(360,20);
RA_x_ms= zeros(360,20);
Z_ms_dyn=  zeros(1,20);
Z_ls_dyn=  zeros(1,20);
M_a_ls= zeros(1,20);
M_m_ls= zeros(1,20);
Main_M_ls= zeros(360,20);
R_bo_y= zeros(360,20);
R_bi_y= zeros(360,20);
R_bo_x= zeros(360,20);
R_bi_x= zeros(360,20);
% %----------------------Calculate Forces/Moments------------------------
for i=1:1:20
z=3.55*i;
[ Ma, Mm, Ta, Tm, R_A_y, R_A_x, ~, ReactionMag ] =...
    main_shaft_dynamics(W(1), W(2), W(3), D(10), D(11), D(15), D(16), D(13), z);
M_a_ms(i)=Ma;
M_m_ms(i)=Mm;
T_a_ms(i)= Ta;
T_m_ms(i)=Tm;
    for j=1:1:360
    RA_y_ms(j,i)=R_A_y(j);
    RA_x_ms(j,i)=R_A_x(j);
    end
Z_ms_dyn(i)=z;
end

for i=1:1:20
z=.321*i;
[ Ma, Mm, Rboy, Rbiy, Rbox, Rbix, Rbomax, Rbimax] =...
    little_shaft_dynamics(W(4), W(5), RA_y_ms(:,1), RA_x_ms(:,1), D(7) , D(8), D(9), D(2), z);
M_a_ls(i)=Ma;
M_m_ls(i)=Mm;
    for j=1:1:360
    R_bo_y(j,i)=Rboy(j);
    R_bi_y(j,i)=Rbiy(j);
    R_bo_x(j,i)=Rbox(j);
    R_bi_x(j,i)=Rbix(j);
    end
Z_ls_dyn(i)=z;
end

%use these for fastener calculations
R_bo_ya = abs((max(R_bo_y) - min(R_bo_y))/2);
R_bo_ym = (max(R_bo_y) + min(R_bo_y))/2 ;
R_bi_ya = abs((max(R_bi_y) - min(R_bi_y))/2);
R_bi_ym = (max(R_bi_y) + min(R_bi_y))/2 ;

disp(ReactionMag);
disp(Rbimax);
disp(Rbomax);
```

```
%%

%-----------------------Plot Forces/Moments------------------------

figure
subplot(4,1,1);
plot(Z_ms_dyn,M_a_ms);
title('Main Shaft - Alternating Moment over z axis')
subplot(4,1,2);
plot(Z_ms_dyn,M_m_ms);
title('Main Shaft - Midrange Moment over z axis')
subplot(4,1,3);
plot(Z_ms_dyn,T_a_ms);
title('Main Shaft - Alternating Torque over z axis')
subplot(4,1,4);
plot(Z_ms_dyn,T_m_ms);
title('Main Shaft - Midrange Torque over z axis')

figure
subplot(2,1,1);
plot(Z_ls_dyn,M_a_ls);
title('Little Shaft - Alternating Moment over z axis')
xlabel('Z Position (in)')
ylabel('\sigma_{alt} (psi)')
subplot(2,1,2);
plot(Z_ls_dyn,M_m_ls);
title('Little Shaft - Midrange Moment over z axis')
xlabel('Z Position (in)')
ylabel('\sigma_{mid} (psi)')
%
%-----------------------Calculate Stresses-------------------------


%---------------------------MAIN SHAFT------------------------------
%Critical Point on Main Shaft:
%CP1 = Connection of the bird beak SHS T joint
%CP2 = Center of Shaft - Inside Location
%CP3 = Hole on edge of Shaft
%------------------------Critical Point 1--------------------------
z_CP1=D(19);
%Stress Concentration Factors
Kt_axe_CP1=1;
%Kt_bend_CP1=1.8;% - for no fillet
Kt_bend_CP1=1.6;
Kt_s_CP1=1;
Sut=65;
Sy=50;

q_CP1=0.8; %double check this taken from slide notes
qs_CP1=0.8; %taken from paper

Kf_axe_CP1=1+q_CP1*(Kt_axe_CP1-1);
Kf_bend_CP1=1+q_CP1*(Kt_bend_CP1-1);
Kf_s_CP1=1+qs_CP1*(Kt_s_CP1-1);

[Ma_CP1_ms, Mm_CP1_ms, Ta_CP1_ms, Tm_CP1_ms, ~, ~, ~] =...
    main_shaft_dynamics(W(1), W(2), W(3), D(10), D(11), D(15), D(16), D(13), z_CP1);
% %alternating
%USE FOR SQUARE SHAFT
[s_axe_alt_ms_CP1,s_bend_alt_ms_CP1,tau_alt_ms_CP1] =
stress_thin_wall_square_shaft(D(17),D(18),0,Ma_CP1_ms,Ta_CP1_ms);
%USE FOR CIRCLE SHAFT
%[s_axe_alt_ms_CP1,s_bend_alt_ms_CP1,tau_alt_ms_CP1] =
stress_cylinder(D(17),D(18),0,Ma_CP1_ms,Ta_CP1_ms);

% %midrange
%USE FOR SQUARE SHAFT
[s_axe_mid_ms_CP1,s_bend_mid_ms_CP1,tau_mid_ms_CP1] =
stress_thin_wall_square_shaft(D(17),D(18),0,Mm_CP1_ms,Tm_CP1_ms);
%USE FOR CIRCLE SHAFT
```

```
%[s_axe_mid_ms_CP1,s_bend_mid_ms_CP1,tau_mid_ms_CP1] =
stress_cylinder(D(17),D(18),0,Mm_CP1_ms,Tm_CP1_ms);

% %von mises
[sig_alt_prime_ms_CP1, sig_mid_prime_ms_CP1]
=von_mises_stresses(s_bend_alt_ms_CP1,s_bend_mid_ms_CP1,...
    tau_alt_ms_CP1,tau_mid_ms_CP1,...
    s_axe_alt_ms_CP1,s_axe_mid_ms_CP1,...
    Kf_axe_CP1,Kf_bend_CP1,Kf_s_CP1);

%Calculate the Endurance Limit
%Se_CP1 = calculate_Se(Sut,1, 1, 1, D(17),1,1);
Se_CP1= calculate_Se(Sut,1, 1, 2, 1,D(17),D(18));

%fatigue saftey factor
SF_fatigue_CP1_ms =
fatigue_safety_factor_mod_goodman(Se_CP1,Sut,(sig_alt_prime_ms_CP1/1000),(sig_mid_prime_ms_CP1/10
00))

%static safety factor
SF_yield_CP1_ms = safety_factor_static(Sy,(sig_alt_prime_ms_CP1/1000),
(sig_mid_prime_ms_CP1/1000))

%plot time respose
%bending stress
figure
t = linspace(0,2*pi,100);
y = s_bend_alt_ms_CP1*sin(t)+s_bend_mid_ms_CP1;
plot(t,y);
title('Time Response for Bending Stress')
xlabel('Time (s)')
ylabel('\sigma (psi)')

%torsion
figure
t = linspace(0,2*pi,100);
y = tau_alt_ms_CP1*sin(t)+tau_mid_ms_CP1;
plot(t,y);
title('Time Response for Shear Stress')
xlabel('Time (s)')
ylabel('\tau (psi)')
%

% % %------------------------Critical Point 2---------------------------
%CP2 = Center of Shaft
z_CP2=D(13)/2;
% % %Stress Concentration Factors
Kt_axe_CP2=1;
Kt_bend_CP2=1;
Kt_s_CP2=1.25;
Sut=65;
Sy=50;

q_CP2=0; %double check this taken from slide notes
qs_CP2=0; %taken from paper

Kf_axe_CP2=1+q_CP2*(Kt_axe_CP2-1);
Kf_bend_CP2=1+q_CP2*(Kt_bend_CP2-1);
Kf_s_CP2=1+qs_CP2*(Kt_s_CP2-1);

[Ma_CP2_ms, Mm_CP2_ms, Ta_CP2_ms, Tm_CP2_ms, ~, ~, ~] =...
    main_shaft_dynamics(W(1), W(2), W(3), D(10), D(11), D(15), D(16), D(13), z_CP2);
% %alternating
[s_axe_alt_ms_CP2,s_bend_alt_ms_CP2,tau_alt_ms_CP2] =
stress_thin_wall_square_shaft(D(17),D(18),0,Ma_CP2_ms,Ta_CP2_ms);
% %midrange
[s_axe_mid_ms_CP2,s_bend_mid_ms_CP2,tau_mid_ms_CP2] =
stress_thin_wall_square_shaft(D(17),D(18),0,Mm_CP2_ms,Tm_CP2_ms);
% %von mises
[sig_alt_prime_ms_CP2, sig_mid_prime_ms_CP2]
=von_mises_stresses(s_bend_alt_ms_CP2,s_bend_mid_ms_CP2,...
```

```
    tau_alt_ms_CP2,tau_mid_ms_CP2,...
    s_axe_alt_ms_CP2,s_axe_mid_ms_CP2,...
    Kf_axe_CP2,Kf_bend_CP2,Kf_s_CP2);

%Calculate the Endurance Limit
Se_CP2 = calculate_Se(Sut,1, 1, 2, 1,D(17),D(18));

%fatigue saftey factor
SF_fatigue_CP2_ms =
fatigue_safety_factor_mod_goodman(Se_CP2,Sut,(sig_alt_prime_ms_CP2/1000),(sig_mid_prime_ms_CP2/10
00))

%static safety factor
SF_yield_CP2_ms = safety_factor_static(Sy,(sig_alt_prime_ms_CP2/1000),
(sig_mid_prime_ms_CP2/1000))

%%
% % % %------------------------Critical Point 3---------------------------
%CP3 = Hole on edge of Shaft
z_CP3=D(20);
% % %Stress Concentration Factors
Kt_axe_CP3=1;
Kt_bend_CP3=5;
Kt_s_CP3=3.5;
Sut=65;
Sy=50;

q_CP3=0.8; %double check this taken from slide notes
qs_CP3=0.8; %taken from paper

Kf_axe_CP3=1+q_CP3*(Kt_axe_CP3-1);
Kf_bend_CP3=1+q_CP3*(Kt_bend_CP3-1);
Kf_s_CP3=1+qs_CP3*(Kt_s_CP3-1);

[Ma_CP3_ms, Mm_CP3_ms, Ta_CP3_ms, Tm_CP3_ms, ~, ~, ~] =...
    main_shaft_dynamics(W(1), W(2), W(3), D(10), D(11), D(15), D(16), D(13), z_CP3);
% %alternating
[s_axe_alt_ms_CP3,s_bend_alt_ms_CP3,tau_alt_ms_CP3] =
stress_thin_wall_square_shaft(D(17),D(18),0,Ma_CP3_ms,Ta_CP3_ms);
% %midrange
[s_axe_mid_ms_CP3,s_bend_mid_ms_CP3,tau_mid_ms_CP3] =
stress_thin_wall_square_shaft(D(17),D(18),0,Mm_CP3_ms,Tm_CP3_ms);
% %von mises
[sig_alt_prime_ms_CP3, sig_mid_prime_ms_CP3]
=von_mises_stresses(s_bend_alt_ms_CP3,s_bend_mid_ms_CP3,...
    tau_alt_ms_CP3,tau_mid_ms_CP3,...
    s_axe_alt_ms_CP3,s_axe_mid_ms_CP3,...
    Kf_axe_CP3,Kf_bend_CP3,Kf_s_CP3);

%Calculate the Endurance Limit
Se_CP3 = calculate_Se(Sut,1, 1, 2, 1,D(17),D(18));

%fatigue saftey factor
SF_fatigue_CP3_ms =
fatigue_safety_factor_mod_goodman(Se_CP3,Sut,(sig_alt_prime_ms_CP3/1000),(sig_mid_prime_ms_CP3/10
00))

%static safety factor
SF_yield_CP3_ms = safety_factor_static(Sy,(sig_alt_prime_ms_CP3/1000),
(sig_mid_prime_ms_CP3/1000))
%%

%---------------------------LITTLE SHAFT------------------------------
%Critical Point on Main Shaft:
%CP4 = Small Diameter Change (Step) closest to the inner bearing
%CP5 = Location of Maximum Moment (Center of where inner bearing sits)
%CP6 = Large Diameter Change after the outer bearing
%------------------------Critical Point 4---------------------------
z_CP4=D(21);
% % %Stress Concentration Factors
%Fatigue Factor=23.0350 Yield Factor=24.4258
```

```
Kt_axe_CP4=1;
Kt_bend_CP4=2;
%Kt_bend_CP4=1;
Sut_ls=78.3;
Sy_ls=60.2;

q_CP4=0.8;


Kf_axe_CP4=1+q_CP4*(Kt_axe_CP4-1);
Kf_bend_CP4=1+q_CP4*(Kt_bend_CP4-1);

[ Ma_CP4_ls, Mm_CP4_ls,~, ~, ~, ~, ~, ~ ] = little_shaft_dynamics(W(4), W(5), RA_y_ms(:,1),
RA_x_ms(:,1), D(7) , D(8), D(9), D(2), z_CP4);
% %alternating
[s_axe_alt_ls_CP4,s_bend_alt_ls_CP4,~] = stress_cylinder(D(23),D(24),0,Ma_CP4_ls,0);
% %midrange
[s_axe_mid_ls_CP4,s_bend_mid_ls_CP4,~] = stress_cylinder(D(23),D(24),0,Mm_CP4_ls,0);
% %von mises
[sig_alt_prime_ls_CP4, sig_mid_prime_ls_CP4]
=von_mises_stresses(s_bend_alt_ls_CP4,s_bend_mid_ls_CP4,...
    0,0,...
    s_axe_alt_ls_CP4,s_axe_mid_ls_CP4,...
    Kf_axe_CP4,Kf_bend_CP4,1);

%Calculate the Endurance Limit
Se_CP4 = calculate_Se(Sut_ls,1, 1, 1, D(23),1,1);

%fatigue saftey factor
SF_fatigue_CP4_ls =
fatigue_safety_factor_mod_goodman(Se_CP4,Sut_ls,(sig_alt_prime_ls_CP4/1000),(sig_mid_prime_ls_CP4
/1000))

%static safety factor
SF_yield_CP4_ls = safety_factor_static(Sy_ls,(sig_alt_prime_ls_CP4/1000),
(sig_mid_prime_ls_CP4/1000))

%%
% %------------------------Critical Point 5---------------------------
%CP5 = Location of Maximum Moment (Center of where inner bearing sits)
%Fatigue Factor=23.0350 Yield Factor=24.4258
z_CP5=D(8);
% % %Stress Concentration Factors
Kt_axe_CP5=1;
Kt_bend_CP5=1;
Sut_ls=78.3;
Sy_ls=60.2;

q_CP5=0;


Kf_axe_CP5=1+q_CP5*(Kt_axe_CP5-1);
Kf_bend_CP5=1+q_CP5*(Kt_bend_CP5-1);

[ Ma_CP5_ls, Mm_CP5_ls,~, ~, ~, ~, ~, ~ ] = little_shaft_dynamics(W(4), W(5), RA_y_ms(:,1),
RA_x_ms(:,1), D(7) , D(8), D(9), D(2), z_CP5);
% %alternating
[s_axe_alt_ls_CP5,s_bend_alt_ls_CP5,~] = stress_cylinder(D(25),D(24),0,Ma_CP5_ls,0);
% %midrange
[s_axe_mid_ls_CP5,s_bend_mid_ls_CP5,~] = stress_cylinder(D(25),D(24),0,Mm_CP5_ls,0);
% %von mises
[sig_alt_prime_ls_CP5, sig_mid_prime_ls_CP5]
=von_mises_stresses(s_bend_alt_ls_CP5,s_bend_mid_ls_CP5,...
    0,0,...
    s_axe_alt_ls_CP5,s_axe_mid_ls_CP5,...
    Kf_axe_CP5,Kf_bend_CP5,1);

%Calculate the Endurance Limit
Se_CP5 = calculate_Se(Sut_ls,1, 1, 1, D(25),1,1);

%fatigue saftey factor
```

```
SF_fatigue_CP5_ls =
fatigue_safety_factor_mod_goodman(Se_CP5,Sut_ls,(sig_alt_prime_ls_CP5/1000),(sig_mid_prime_ls_CP5
/1000))

%static safety factor
SF_yield_CP5_ls = safety_factor_static(Sy_ls,(sig_alt_prime_ls_CP5/1000),
(sig_mid_prime_ls_CP5/1000))
%plot time respose
%bending stress
figure
t = linspace(0,2*pi,100);
y = s_bend_alt_ls_CP5*sin(t)+s_bend_mid_ls_CP5;
plot(t,y);
title('Time Response for Bending Stress')
xlabel('Time (s)')
ylabel('\sigma (psi)')


%------------------------Critical Point 6---------------------------
%CP6 = Large Diameter Change after the outer bearing
%GIVES HUGE SAFETY FACTOR, AFTER BEARINGS, NOT A CRITICAL POINT
z_CP6=D(26);
% % %Stress Concentration Factors
Kt_axe_CP6=1;
Kt_bend_CP6=2.6;
%Kt_bend_CP6=1;
Sut_ls=78.3;
Sy_ls=60.2;

q_CP6=0.8;


Kf_axe_CP6=1+q_CP6*(Kt_axe_CP6-1);
Kf_bend_CP6=1+q_CP6*(Kt_bend_CP6-1);

[ Ma_CP6_ls, Mm_CP6_ls,~, ~, ~, ~, ~, ~ ] = little_shaft_dynamics(W(4), W(5), RA_y_ms(:,1),
RA_x_ms(:,1), D(7) , D(8), D(9), D(2), z_CP6);
% %alternating
[s_axe_alt_ls_CP6,s_bend_alt_ls_CP6,~] = stress_cylinder(D(27),D(24),0,Ma_CP6_ls,0);
% %midrange
[s_axe_mid_ls_CP6,s_bend_mid_ls_CP6,~] = stress_cylinder(D(27),D(24),0,Mm_CP6_ls,0);
% %von mises
[sig_alt_prime_ls_CP6, sig_mid_prime_ls_CP6]
=von_mises_stresses(s_bend_alt_ls_CP6,s_bend_mid_ls_CP6,...
    0,0,...
    s_axe_alt_ls_CP6,s_axe_mid_ls_CP6,...
    Kf_axe_CP6,Kf_bend_CP6,1);

%Calculate the Endurance Limit
Se_CP6 = calculate_Se(Sut_ls,1, 1, 1, D(27),1,1);

%fatigue saftey factor
SF_fatigue_CP6_ls =
fatigue_safety_factor_mod_goodman(Se_CP6,Sut_ls,(sig_alt_prime_ls_CP6/1000),(sig_mid_prime_ls_CP6
/1000))

%static safety factor
SF_yield_CP6_ls = safety_factor_static(Sy_ls,(sig_alt_prime_ls_CP6/1000),
(sig_mid_prime_ls_CP6/1000))

%%
% %--------------------BEARINGS---------------
% %Inner Bearing
% IB_SF = bearing(Rbimax,1)
% %Outer Bearing
% OB_SF = bearing(Rbomax,1)
% disp(Rbimax);
% disp(Rbomax);
%%
% %-------------Forces on Bolted Joints----------
% [ F_ot_y, F_it_y ] = FBD_TriangleAttachments(Rbomax, Rbimax, W(6));
```

```
% [ Rx, Ry, Rz, F_leg_x, F_leg_y, F_leg_z ] = FBD_Leg( F_ot_y, F_it_y, W(7), D(4), D(5), D(6));
% Fonleg_Max=(F_leg_x^2+F_leg_y^2)^(1/2)
```

9. Main_shaft_dynamics

```
function [ Ma, Mm, Ta, Tm, R_A_y, R_A_x, M, ReactionMag ] = main_shaft_dynamics(W_ms, W_cw, W_B,
z_cw1, z_cw2, r_B, r_cw, L, z)
%UNTITLED Summary of this function goes here
%   inputs: weights, moment arms, shaft length,and z
%   z =the point you where want to calculate the moment. point C = (z=o, origin)
%   outputs: Ma, Mm, T_a, T_m, R_A_y (array), R_A_x (array), M (array) (no axial, R_A_z assumed
zero)
%
RPM=60;
omega = RPM*2*pi/60 ; % rev/min to rad/s

% Centripetal forces
C_B= r_B*W_B*omega^2/(32.17*12);
C_cw= r_cw*W_cw*omega^2/(32.17*12);

R_A_y= zeros(360,1);
R_A_x= zeros(360,1);
M= zeros(360,1);
for i = 1:1:360

    theta_d=i;
    theta_r = theta_d*pi/180;

    % Reaction forces
    R_A_y(i)= ((C_cw - C_B)*(L/2)*cos(theta_r) + z_cw1*(W_cw/2) + z_cw2*(W_cw/2)+ (L/2)*(W_B
+W_ms))/L ;
    R_C_y= R_A_y(i);

    R_A_x(i)= (C_B - C_cw)*sin(theta_r)/2 ;
    R_C_x= R_A_x(i);

    % If statements based on z, for moment diagram
    if z < z_cw1
        M_x = -R_C_y*z ;
        M_y = R_C_x*z;
    else
        if z < L/2
            M_x = -R_C_y*z + (W_cw/2)*(z-z_cw1);
            M_y = R_C_x*z;
        else
            if z < z_cw2
                M_x = -R_C_y*z + (W_cw/2)*(z-z_cw1) + ((C_cw - C_B)*cos(theta_r) + (W_B
+W_ms))*(z-L/2);
                M_y = R_C_x*z + (C_cw -C_B)*(z-L/2)*sin(theta_r);
            else
                M_x = -R_C_y*z + (W_cw/2)*(z-z_cw1) + ( (C_cw - C_B)*cos(theta_r) + (W_B +W_ms)
)*(z-L/2) + (W_cw/2)*(z-z_cw2);
                M_y = R_C_x*z + (C_cw -C_B)*(z-L/2)*sin(theta_r);
            end
        end
    end


    % Moment Magnitude
    M(i) = (M_x^2 +M_y^2)^(1/2);

    % Torque
    T(i) = (W_B*r_B - W_cw*r_cw)*sin(theta_r);

    %Reaction force magnitude
    RM(i) = (R_C_y^2 +R_C_x^2)^(1/2);
end

Ma = abs((max(M) - min(M))/2);
Mm = (max(M) + min(M))/2 ;
```

```
Ta = abs((max(T) - min(T))/2);
Tm = (max(T) + min(T))/2 ;

ReactionMag=max(RM);


end
```

10. Master Code

```
    x = input('x from shaft top [max 6.00]: ');
%%
%------------------------GEOMETRY------------------------------
l1=2.56;
l2=5.13;
l3=6;
r=1.25;
Area=pi*r^2;
%---------------------Weight and RPM---------------------------
W=460; % lbf
w=60*(2*pi)/60; %rad/s
%%
%OBTAINING FORCES
Rxn_Forces()
[At,Bt,Fbolt]=FBD1_Sec_Shaft(Rn,Rt,Mr,l1,l2);
[A_max]=max(At);
[B_max]=max(Bt);

%%OBTAINING STRESSES AND MOMENT
[SA,SB,ST,M]=Stress_Sec_Shaft1(At,Bt,Rn,Rt,Mr,x,l1,l2,l3,Area);
%%OBTAINING MAX,MIN And MID
MaxMinMid()
%%OBTAINING STRESSES AND MOMENT AT X
[SAx,SBx,STx,Mx]=Stress_Sec_Shaft(A_max,B_max,Rn,Rt,Mr,x,l1,l2,l3,Area);

%%
%------------------------STRESS CONCENTRATIONS----------------------------
if (x==l1)
    Kfa=1.05;
    Kfb=1.05;
elseif (x==l2)
    Kfa=4.5;
    Kfb=4.5;
else
    Kfa=1;
    Kfb=1;
end
Kft=1;

%%
%----------------------RESULTS--------------------------------------------

%%FATIGUE ANALYSIS
Sut=67.45;
Sy=33.4;
[Se]=calculate_Se(Sut,1,1,0,2,0,0);
[sig_a_p,sig_m_p]=von_mises_stresses(sig_a_b,sig_m_b,tau_a_t,tau_m_t,sig_a_ax,sig_m_ax,Kfb,Kft,Kf
a);
[sf_fatigue]=fatigue_safety_factor_mod_goodman(Se,Sut,sig_a_p,sig_m_p);
[sf_static]=safety_factor_static(Sy,sig_a_p,sig_m_p);
fprintf('SAFETY FACTOR:%d\n',sf_fatigue);
fprintf('STATIC S.FACTOR:%d\n',sf_static);

%%BEARINGS ANALYSIS
fprintf('Moment:%f\n',Mx);
fprintf('Axial force:%f\n',SAx*Area);
fprintf('Bending Stress:%f\n',SBx);
fprintf('Axial Stress:%f\n',SAx);
fprintf('Alternating Von Stress:%f\n',sig_a_p);
fprintf('Median Von Stress:%f\n',sig_m_p);
```

```
%%
%--------------------EXCEL-----------------------------------
filename = 'testdata.xlsx';
A = [x;Area;Mx;SAx*Area;SBx;SAx;sig_a_p;sig_m_p;sf_static;sf_fatigue];
xlswrite(filename,A)

%%
%--------------------PLOTS-----------------------------------
%{
figure(1)
plot(t,Rn,t,Rt)
title('Reaction Forces at Secondary Shaft')
legend('Rn','Rt')
xlabel('time(s)')
ylabel('lbf')


figure(2)
plot(t,Mr)
title('Reaction Moment at Secondary Shaft')
xlabel('time(s)')
ylabel('psi')
%}
figure(3)
plot(t,SA,t,SB,t,ST)
legend('axial stress','bending stress','torsional stress')
grid on
```

11. Rxn_forces

```
%%REACTION FORCES Rn, Rt, Mr due to the circulation of the
%Center of mass (see Free Body Diagram)

%%GEOMETRY AND WEIGHT
m=460;  %lbf
g=32.2; %ft/s^2
w=60*(2*pi)/60;    %rad/s   %50 rev/min
Rg=64/12;  %feet
t=0:0.01:2*pi;

%% Reaction Forces and Moment at Shaft

Rn=(-((m/32.2)*(w^2)*Rg)+(m*sin(w*t)))/sqrt(2);
Rt=((m*sin(w*t)))/sqrt(2);
Mr=(m*(Rg*12)*sin(w*t))/sqrt(2);
```

12. Safety_factor_static

```
function [sf_static] = safety_factor_static(S_y,sig_a_p, sig_m_p)
%inputs: S_y and von mises sig_a_prime, sig_m_prime
%outputs: asfety factor for static yield at first cycle. conservative
%estimate.

% this commented in line would be for normal static check but unsure what
% sig_a, sig_m, tau_a and tau_m are.
% sig_prime_max= ( (sig_a + sig_m)^2 +3*(tau_a + tau_m)^2 )^(1/2);

%conservative estimate
sig_max_p= sig_a_p + sig_m_p;

sf_static=S_y/sig_max_p;

end
```

13. Stress_cylinder

```
function [ axial,bending,torsion] = stress_cylinder(do,di,F,M,T)
%inputs: radius, Force, Moment, Torque
%outputs: axial stress, bending stress and torsional shear stress
%neglecting transverse shear
%bending stress is for the top of the beam
```

```
A= (pi*(do-di)/4)^2;
axial=F/A;
bending = 32*M/(pi*(do-di)^3);
torsion= 16*T/(pi*(do-di)^3);

end
```

14. Stress_Sec_Shaft1

```
function [SA,SB,ST,M] = Stress_Sec_Shaft1( At,Bt,Fn,Ft,Mr,x,l1,l2,l3,Area1)
Area=Area1;
d=sqrt(4*Area/pi);

if (x>l3)
   printf('wrong entry')
end

if (x<=l1)
    M=Ft*x-Mr;
    SB = 32*M/(pi*d^3);
    SA=Fn/Area;
end

if (l1<x)&&(x<=l2)
    M=Ft*x+At*(x-l1)-Mr;
    SB = 32*M/(pi*d^3);
    SA=Fn/Area;
end

if (l2<x)&&(x<l3)
    M=Ft*x+At*(x-l1)-Bt*(x-l2)-Mr;
    SB = 32*M/(pi*d^3);
    SA=(Fn)/Area;
end

ST=0;
end
```

15. Stress_Sec_Shaft

```
[SA_max,t_max1]=max(SA);
[SA_min,t_min1]=min(SA);
[SA_mid]=(SA_max+SA_min)/2;

[SB_max,t_max2]=max(SB);
[SB_min,t_min2]=min(SB);
[SB_mid]=(SB_max+SB_min)/2;

[ST_max,t_max3]=max(ST);
[ST_min,t_min3]=min(ST);
[ST_mid]=(ST_max+ST_min)/2;

sig_a_b=SB_max-SB_mid;
sig_m_b=SB_mid;
sig_a_ax=SA_max-SA_mid;
sig_m_ax=SA_mid;
tau_a_t=ST_max-ST_mid;
tau_m_t=ST_mid;

function [SA,SB,ST,M] = Stress_Sec_Shaft( At,Bt,Rn,Rt,Mrr,x,l1,l2,l3,Area)
Area=Area;
d=sqrt(4*Area/pi);
Fn=max(Rn);
Ft=max(Rt);
Mr=max(Mrr);

if (x>l3)
   printf('wrong entry')
end
```

```
if (x<=l1)
    M=Ft*x-Mr;
    SB = 32*M/(pi*d^3);
    SA=Fn/Area;
end

if ((l1<x)&&(x<=l2))
    M=Ft*x+At*(x-l1)-Mr;
    SB = 32*M/(pi*d^3);
    SA=(Fn)/Area;
end

if ((l2<x)&&(x<l3))
    M=Ft*x+At*(x-l1)-Bt*(x-l2)-Mr;
    SB = 32*M/(pi*d^3);
    SA=(Fn)/Area;
end

ST=0;

end
```

16. *stress_thin_wall_square_shaft*

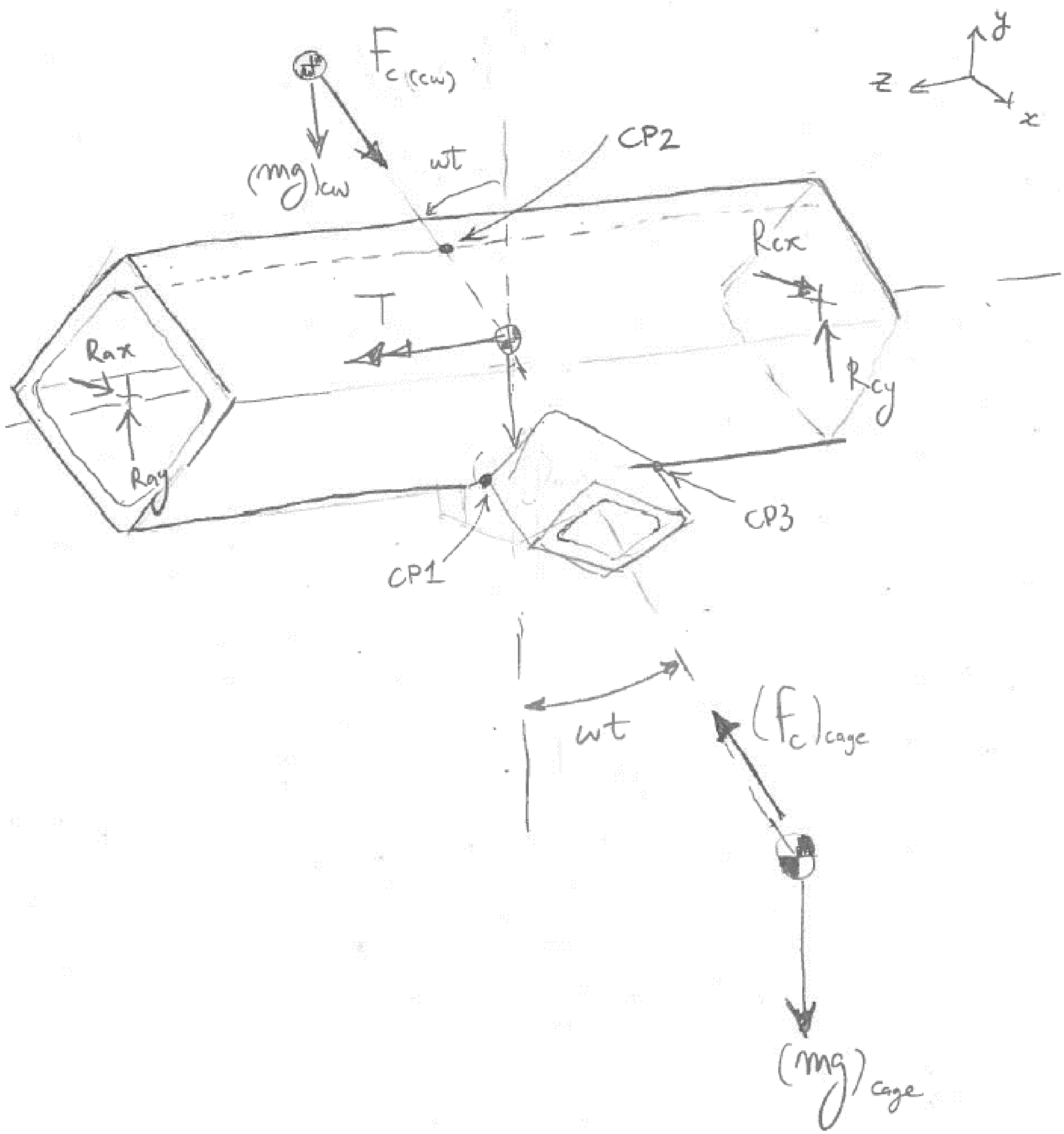
```
function [ axial,bending,torsion_thin] = stress_thin_wall_square_shaft(o_w,i_w,F,M,T)
%inputs: outer width,innner width, Force, Moment, Torque
%outputs: axial stress, bending stress and torsional shear stress
%neglecting transverse shear
%bending stress is for the top of the beam
I=((o_w^4)-(i_w^4))/12;

A= (o_w^2) - (i_w^2); %actual solid area
A_e=i_w^2; %enclosed area (no solid)

t_thin= (o_w-i_w)/2;
t_corner= (t_thin^2 +t_thin^2)^(1/2);
%J=t*((a-2)^2)*((a-t)^2); not necessary for this formula, but kept it here
%in case we switch formulas. (for torsion)

axial=F/A;
bending = M*(o_w/2)/I;

torsion_thin= T/(2*t_thin*A_e);
%torsion_corner=T/(2*t_corner*A_e);%not the max torsion, but location where stress concentration
is

end
```

17. *von_mises_stresses*

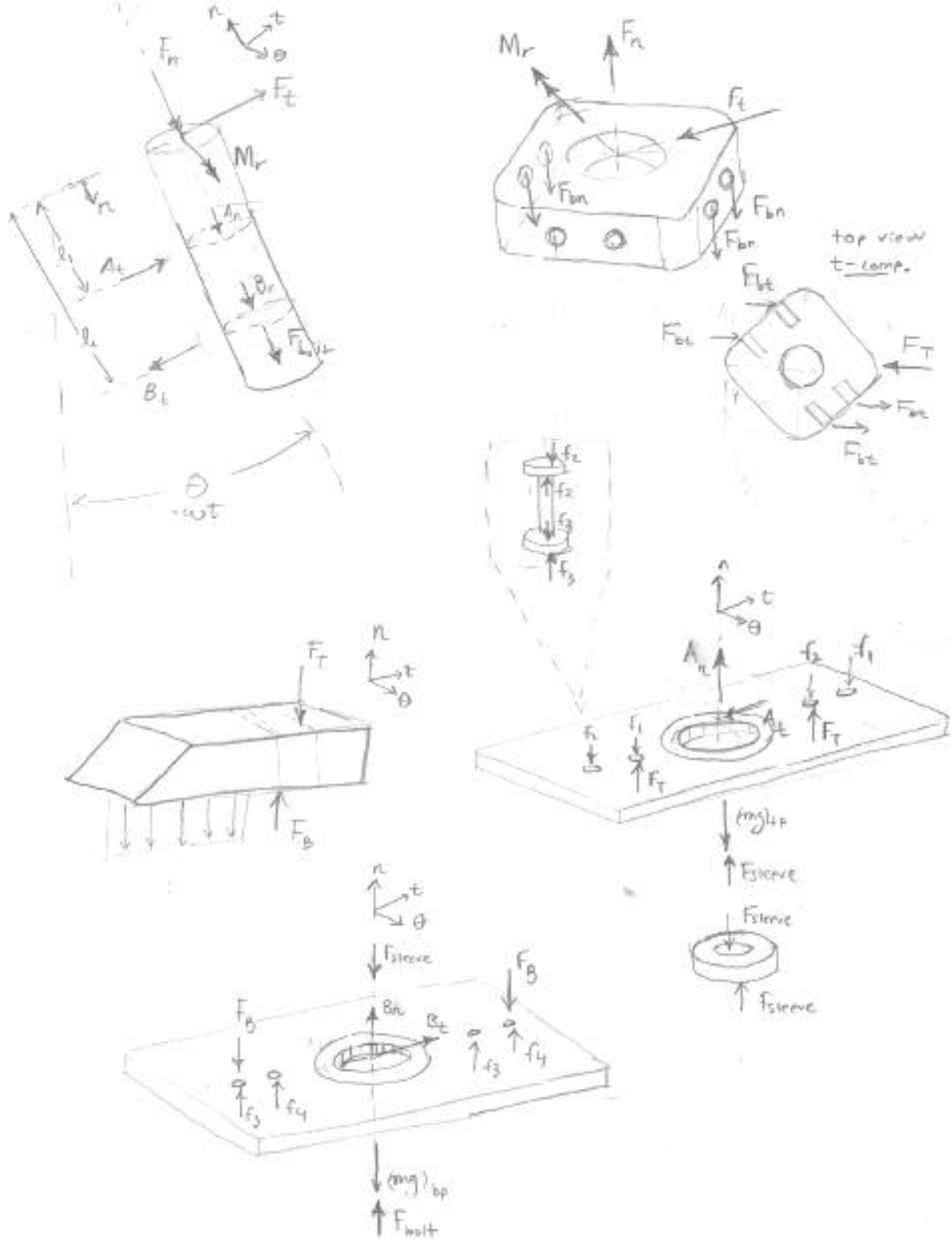
```
function [sig_a_prime, sig_m_prime] =
von_mises_stresses(sig_a_b,sig_m_b,tau_a_t,tau_m_t,sig_a_ax,sig_m_ax, kf_b,kfs_t,kf_ax)
%inputs: stress alternating bending,stress midrange bending,stress
%alternating torsion,stress midrange torsion, stress alternating
%axial,stress midrange axial
%outputs: axial stress, bending stress and torsional shear stress

%calculates von mises for fatigue case. For static case enter values in
%midrange and 0 for everything else.


    sig_a_prime= ( (kf_b*sig_a_b+(kf_ax*sig_a_ax/0.85))^2 + 3*(kfs_t*tau_a_t)^2 )^(1/2);

    sig_m_prime= ( (kf_b*sig_m_b+kf_ax*sig_m_ax)^2 + 3*(kfs_t*tau_m_t)^2 )^(1/2);

end
```

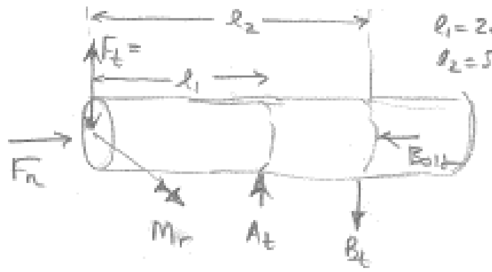
B. Free-Body Diagrams & Miscellanious Sample Calculations

1. Main Shaft



2. Secondary Shaft





$\ell_1 = 2.56$
$\ell_2 = 5.13$

$d = 2''$

$m = 460$ lbf

$\omega = 60 \frac{rev}{min} \times \frac{2\pi}{1 rev} \times \frac{1 min}{60} = 6.28 \frac{rad}{s}$

$F_n = \frac{\left[-\left(\frac{460}{32.2}\right)(6.28)^2\left(\frac{64}{12}\right) + 460 \sin \omega t\right]}{\sqrt{2}}$

$(F_n)_a = 325.3$

$(F_n)_m = -2,126.9$

$F_t = \frac{460 \sin \omega t}{\sqrt{2}}$

$(F_t)_a = 325.3$

$(F_t)_m = 0$

$M_r = \frac{(460)(64) \sin \omega t}{\sqrt{2}}$

$(M_r)_a = 20,817$

$(M_r)_m = 0$

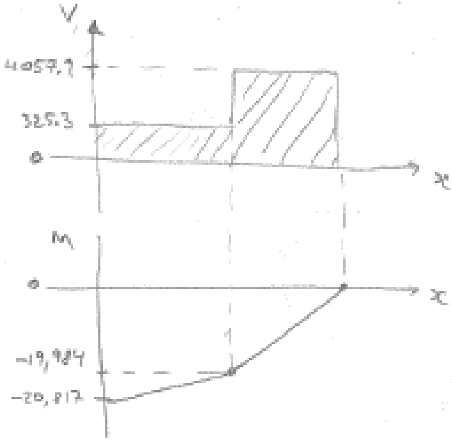
Equilibrium Equation

B/

$\sum M_B = 0 \qquad M_r - F_t \ell_2 - A_t \ell_1 = 0$

$A_t = \frac{M_r - F_t \ell_2}{\ell_1} = 3732.6$ lbf

$\sum F_t = 0 \qquad B_t = F_t + A_t = 4057.9$ lbf



BENDING STRESSES $\qquad \sigma_b = \frac{32 M(x)}{\pi d^3}$

@ ℓ_1: $(\sigma_b)_a = +25,444.4$ psi

$(\sigma_b)_m = 0$

AXIAL STRESSES $\qquad \sigma_{ax} = \frac{F}{A}$

$(\sigma_{ax})_a = \frac{325.3}{\pi(1^2)} = 103.55$

$(\sigma_{ax})_m = \frac{-2126.9}{\pi(1^2)} = -677.0$

VON MISES $K_{fax} = 1.05$ $K_{fb} = 1.05$

$$(\sigma_a)' = \left\{ \left[K_{fb}(\sigma_b)_a + K_{fa} \frac{(\sigma_{ax})_a}{.85} \right]^2 + 3\left[K_{fs} \tau_a \right]^2 \right\}^{1/2}$$

$= 26,844.5 = 26.84$ ksi

$$(\sigma_m)' = \left\{ \left[K_{fb}(\sigma_b)_m + K_{fa}(\sigma_{ax})_m \right]^2 + 3\left[K_{fs} \tau_m \right]^2 \right\}^{1/2}$$

$= 710.85$ psi $= .71085$ ksi

mod - Goodman

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

$n = 0.7327$

S_e :

$S_{ut} = 67.45$ $S_e' = 0.5 \times S_{ut}$ $= 33.73$

FACTORS

a: (for machined)

$k_a = (2.70)(67.45)^{-.265}$

$k_a = 0.8845$

b:

$d = 2''$

$k_b = 0.879(2)^{-.107}$

$k_b = 0.8162$

c: (combined)

$k_c = 1$

$k_d = 1$

$k_e = 0.814$ (for 99%)

$k_f = 1$

$S_e = k_a k_b k_c k_e k_f S_e'$

$S_e = 19.82$ ksi

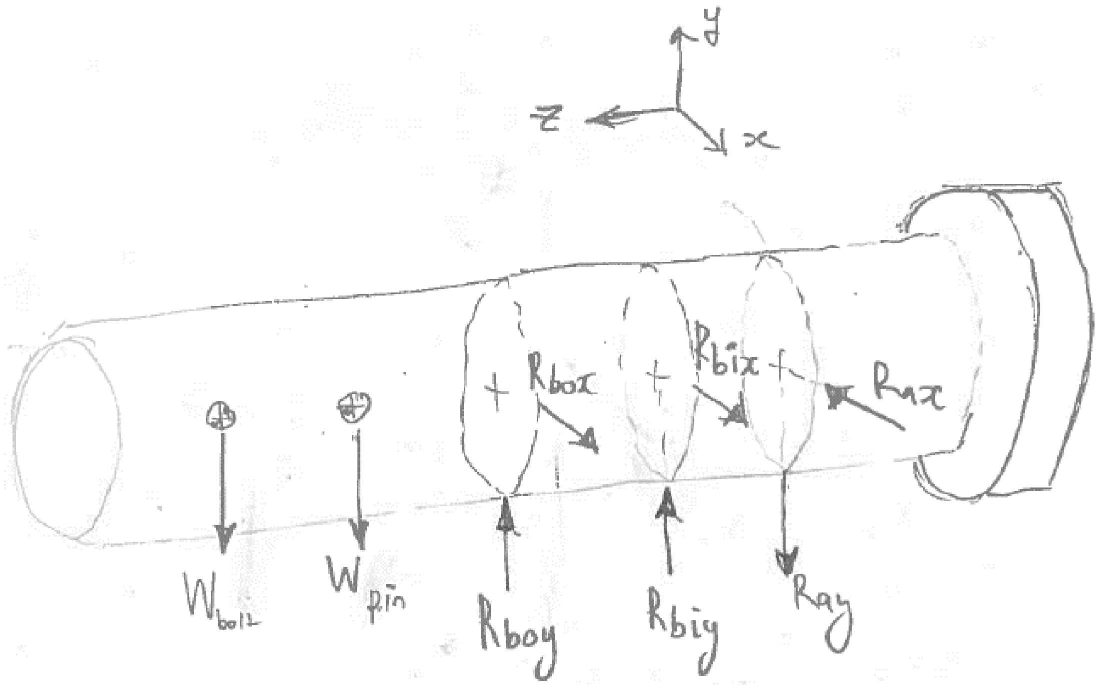
static N $S_y = 33.4$

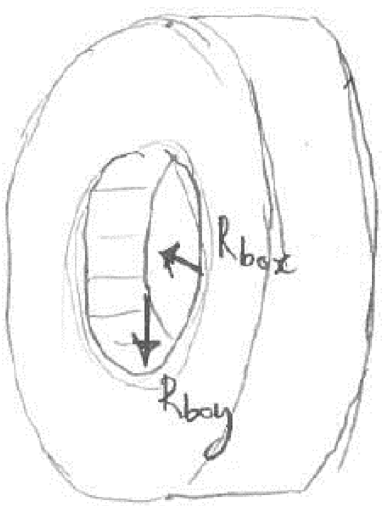
$$n = \frac{S_y}{(\sigma_{ax})_{max}} = \frac{S_y}{(\sigma_a)' + (\sigma_m)'}$$

$n = 1.212$

MATLAB Values	
Distance from top [in]	2.5600
Cross Sectional Area [in^2]	3.1416
Moment [lbf in]	-1.9985e+04
Axial Force [lbf]	-1.8016e+03
Radial Force [lbf]	-2.5445e+04
Stress due to moment [psi]	-573.4744
Stress due to axial force [psi]	2.6845e+04
Von Mises Alternating [psi]	710.8613
Von Mises Middle [psi]	1.2121
Static Safety Factor	0.7325

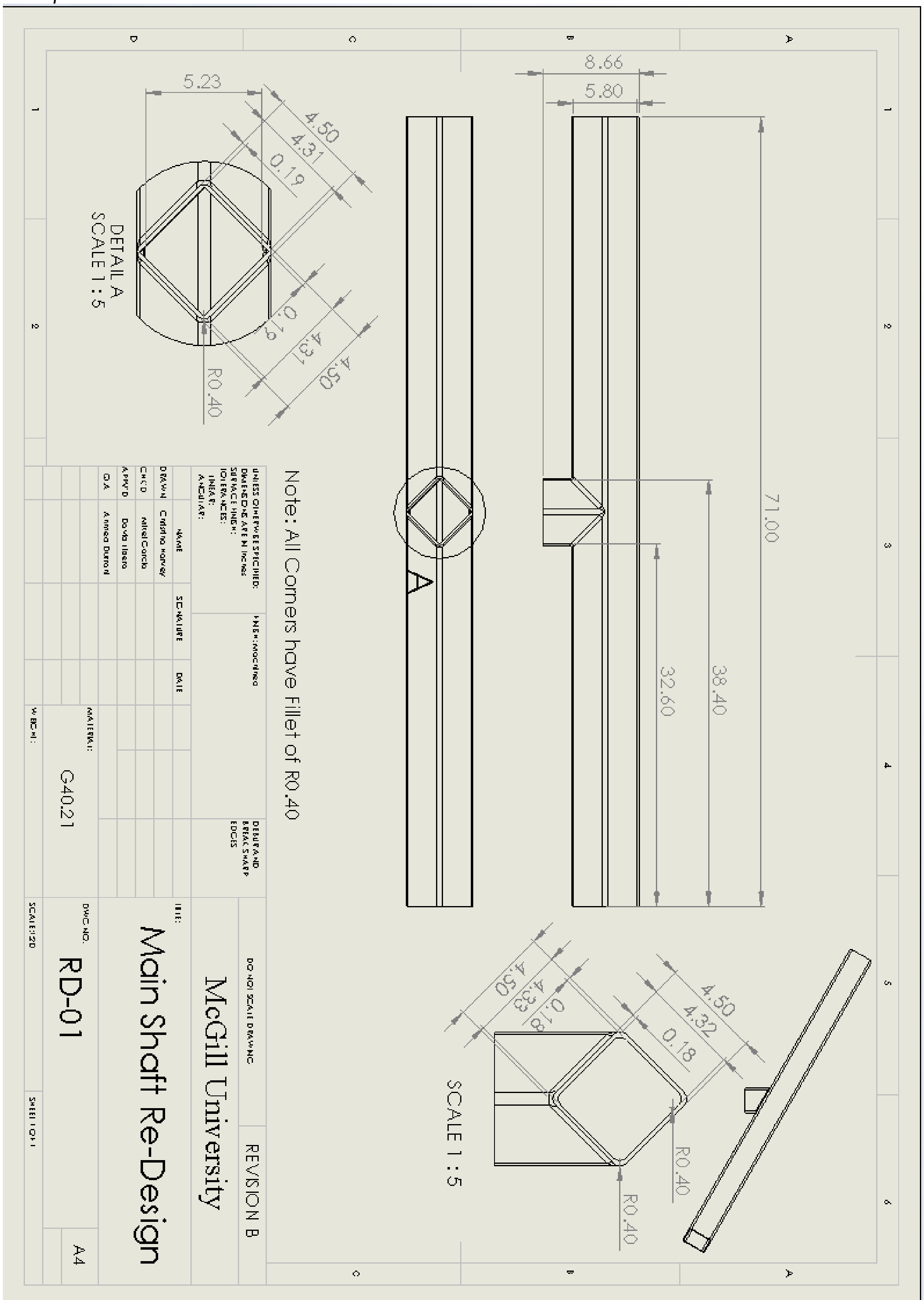
3. Pin and Outer Bearing on Pin



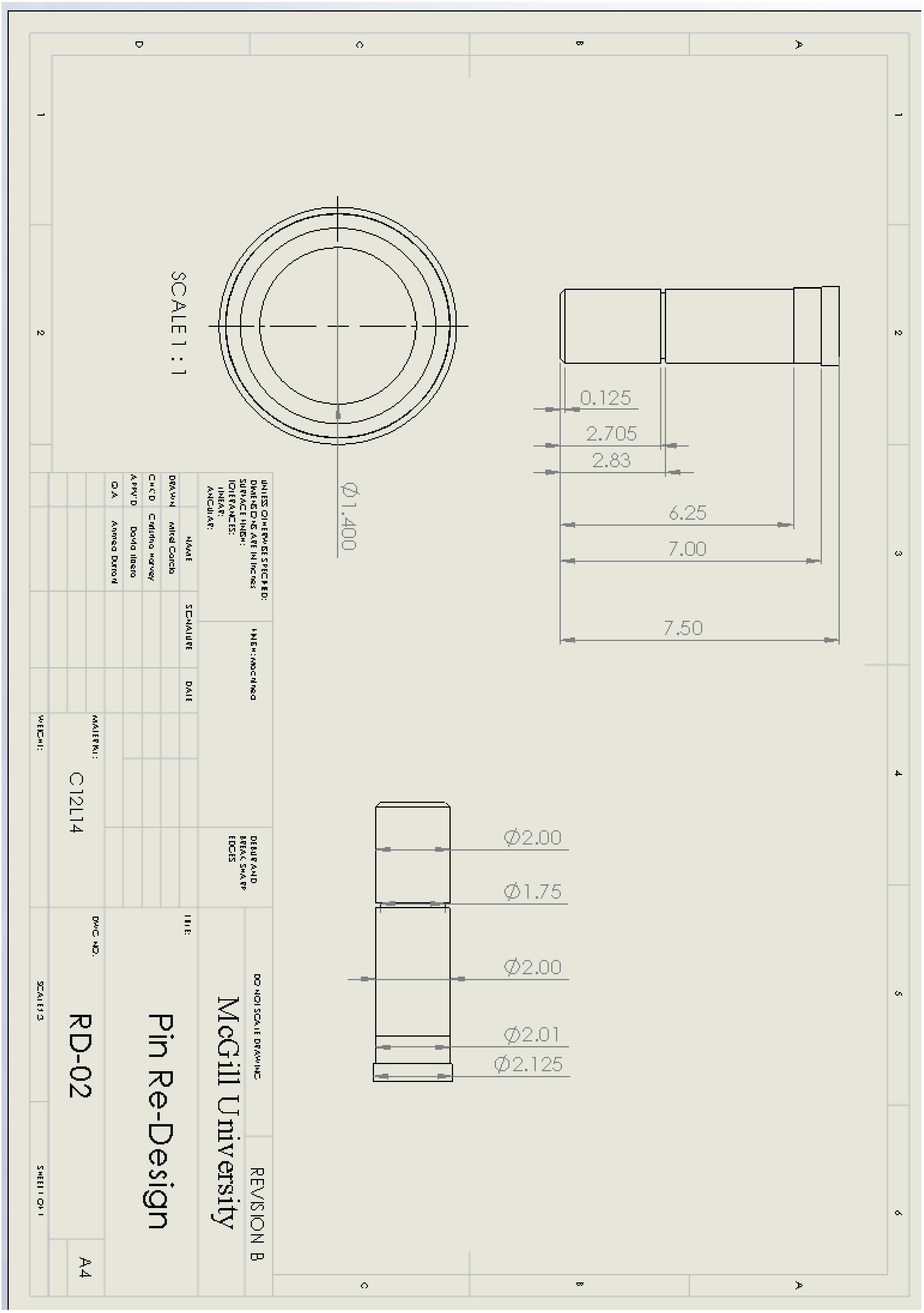


C. Re-Designed Drawings

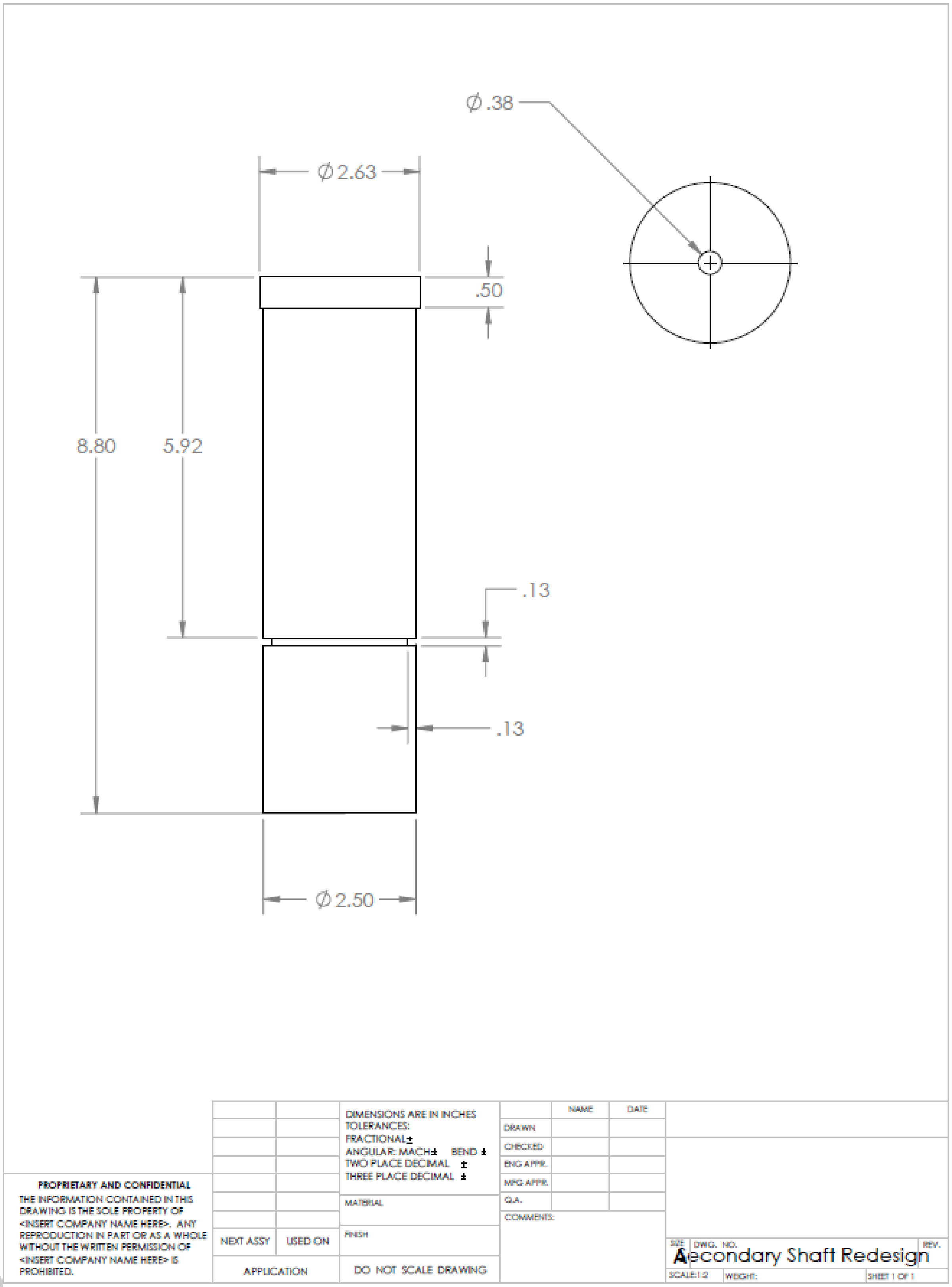
1. Main Shaft



2. Pin

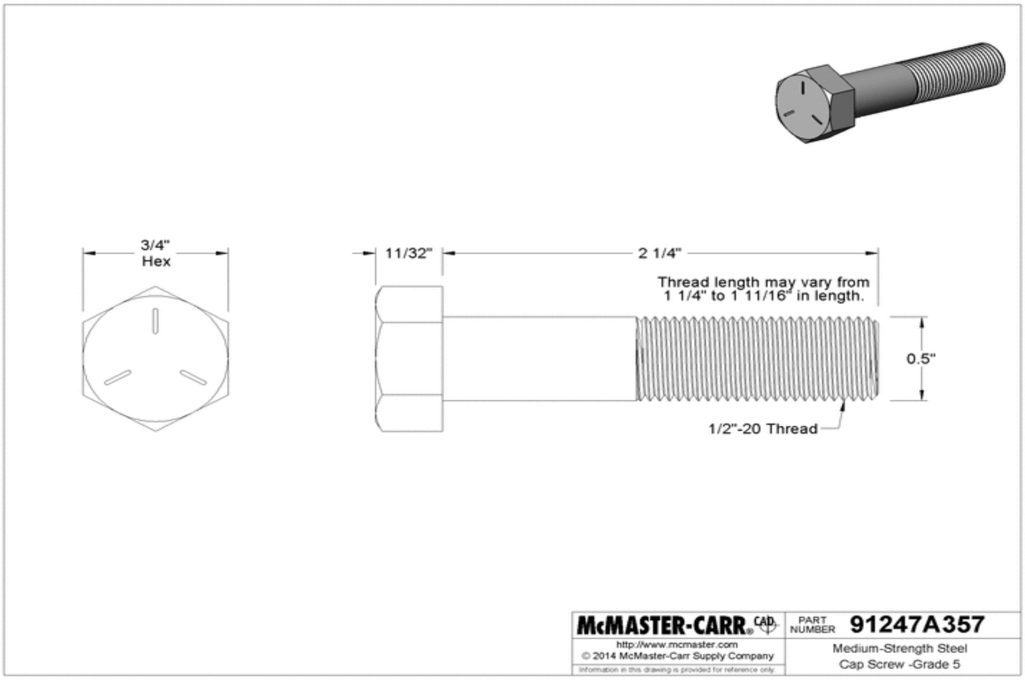

SCALE 1 : 1
0.125
2.705
2.83
6.25
7.00
7.50
Ø1.400
Ø2.00
Ø1.75
Ø2.00
Ø2.01
Ø2.125
McGill University
REVISION B
Pin Re-Design
RD-02
C12L14
A4

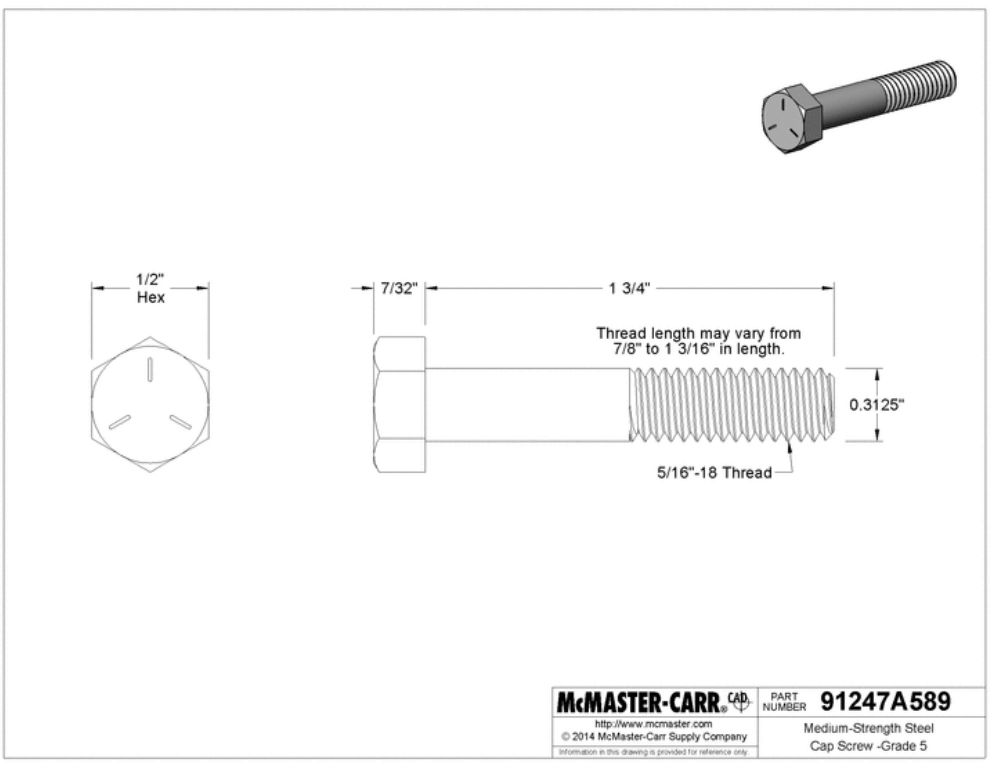
3. Secondary Shaft



4. Bearings

5. Bolted Joints





D. Bill of Materials

Our Material	Name	Main Assy	Individual Assy	Volume (in³)	Density	Weight
ASTM A-283 Gr. D	S-Thing	SX-A05	SX-P66	1.920	0.284	0.545
ASTM A-283 Gr. D	S-Thing	SX-A05	SX-P66	1.920	0.284	0.545
ASTM A-283 Gr. D	S-Thing	SX-A05	SX-P66	1.920	0.284	0.545
ASTM A-283 Gr. D	S-Thing	SX-A05	SX-P66	1.920	0.284	0.545
G40.21	Front Long Tube	SX-A05	SX-P60	44.820	0.284	12.711
G40.21	Front Long Tube	SX-A05	SX-P60	44.820	0.284	12.711
G40.21	Back Bottom Tube	SX-A05	SX-P62	24.400	0.284	6.920
G40.21	Back Bottom Tube	SX-A05	SX-P62	24.400	0.284	6.920
G40.21	Back Mid Tube	SX-A05	-	16.240	0.284	4.606
G40.21	Back Mid Tube	SX-A05	-	16.240	0.284	4.606
G40.21	Back Top Tube	SX-A05	SX-P58	21.280	0.284	6.035
G40.21	Back Top Tube	SX-A05	SX-P58	21.280	0.284	6.035
ASTM A-283 Gr. D	Bottom Fitting	SX-A05	SX-P67	1.470	0.284	0.417
ASTM A-283 Gr. D	Bottom Fitting	SX-A05	SX-P67	1.470	0.284	0.417
ASTM A-283 Gr. D	Bottom Fitting	SX-A05	SX-P67	1.470	0.284	0.417
ASTM A-283 Gr. D	Bottom Fitting	SX-A05	SX-P67	1.470	0.284	0.417
G40.21	Bottom Tube	SX-A05	SX-P63	14.870	0.284	4.217
G40.21	Bottom Support Tubes	SX-A05	SX-P64	2.770	0.284	0.786
G40.21	Bottom Support Tubes	SX-A05	SX-P64	2.770	0.284	0.786
G40.21	Bottom Support Tubes	SX-A05	SX-P64	2.770	0.284	0.786
G40.21	Bottom Support Tubes	SX-A05	SX-P64	2.770	0.284	0.786
Foam	Cushions	-	-	136.090	0.001	0.142
Foam	Cushions	-	-	136.090	0.001	0.142
Foam	Cushions	-	-	332.820	0.001	0.347
Foam	Cushions	-	-	332.820	0.001	0.347
ASTM A-283 Gr. D	Footplate	SX-A06	SX-P70	80.680	0.284	22.913
G40.21	Vertical Square Tube on Top	SX-A03	SX-P56	6.190	0.284	1.756
G40.21	Vertical Square Tube on Top	SX-A03	SX-P56	6.190	0.284	1.756
ASTM A-283 Gr. D	Tiny Coverplate	SX-A03	-	0.520	0.284	0.148
SS304	Tiny Coverplate	SX-A03	-	0.520	0.289	0.150
G40.21	Horizontal Square Tube on Top	SX-A03	SX-P54	19.440	0.284	5.513
G40.21	Horizontal Square Tube on Top	SX-A03	SX-P54	19.440	0.284	5.513
Gr.8.8	Large Nut on Vertical Shaft	SX-A03	-	11.770	0.284	3.343
ASTM A-283 Gr. D	Triangle Brace on Top	SX-A03	SX-P57	3.110	0.284	0.883
ASTM A-283 Gr. D	Triangle Brace on Top	SX-A03	SX-P57	3.110	0.284	0.883
ASTM A-283 Gr. D	Large plate - Contains Bearing	SX-A03	SX-P52	47.680	0.284	13.541
ASTM A-283 Gr. D	Large plate - Contains Bearing	SX-A03	SX-P52	47.680	0.284	13.541
Gr.8.8	Missing Washers	SX-A03	-		0.284	0.000
Gr.8.8	Missing Nuts	SX-A03	-		0.284	0.000

Gr.8.8	Long Bolt - Vertical	SX-A03	-	0.830	0.284	0.236
Gr.8.8	Long Bolt - Vertical	SX-A03	-	0.830	0.284	0.236
Gr.8.8	Short Bolt - Vertical	SX-A03	-	0.840	0.284	0.239
Gr.8.8	Short Bolt - Vertical	SX-A03	-	0.840	0.284	0.239
SS304	Bearing	SX-A03	-	4.520	0.289	1.306
SS304	Bearing	SX-A03	-	4.520	0.289	1.306
6061-T6	Cover Plate	SX-A03	-	11.560	0.098	1.128
Bronze	Insert for Braking System	SX-A03	-	0.120	0.310	0.037
Bronze	Insert for Braking System	SX-A03	-	0.120	0.310	0.037
ASTM A-283 Gr. D	Vertical Shaft	SX-A03	SX-P13	26.750	0.284	7.597
ASTM A-283 Gr. D	Square Attachment	SX-A03	SX-P14	33.560	0.284	9.531
Gr.8.8	Short Bolt on V Shaft x4	-	-	1.480	0.284	0.420
Gr.8.8	Locking Feature x4	-	-	0.240	0.284	0.068
ASTM A-283 Gr. D	Washer x4	-	-	0.160	0.284	0.045
ASTM A-283 Gr. D	Counterweight	SX-A03	SX-P18	404.590	0.284	114.904
ASTM A-283 Gr. D	Counterweight Support	SX-A03	SX-P17	7.470	0.284	2.121
ASTM A-283 Gr. D	Counterweight Support	SX-A03	SX-P17	7.470	0.284	2.121
G40.21	Main Horizontal Shaft	SX-A03	SX-P15	265.480	0.284	75.291
ASTM A-283 Gr. D	small piece attached to leg	SX-A03	SX-P03	34.600	0.284	9.826
ASTM A-283 Gr. D		SX-A03	SX-P03	34.600	0.284	9.826
ASTM A-283 Gr. D		SX-A03	SX-P03	34.600	0.284	9.826
ASTM A-283 Gr. D		SX-A03	SX-P03	34.600	0.284	9.826
ASTM A-283 Gr. D		SX-A03	SX-P03	34.600	0.284	9.826
ASTM A-283 Gr. D	small piece attached to leg	SX-A03	SX-P03	34.600	0.284	9.826
ASTM A-283 Gr. D		SX-A03	SX-P03	34.600	0.284	9.826
ASTM A-283 Gr. D		SX-A03	SX-P03	34.600	0.284	9.826
SS304	Bearing	SX-A03	-	4.520	0.289	1.306
SS304	Bearing	SX-A03	-	4.520	0.289	1.306
SS304	Bearing	SX-A03	-	4.520	0.289	1.306
SS304	Bearing	SX-A03	-	4.520	0.289	1.306
C12L14	little shaft	SX-A03	SX-P13	26.750	0.284	7.597
C12L14	little shaft	SX-A03	SX-P13	26.750	0.284	7.597
ASTM A-283 Gr. D	Square Attachment	SX-A03	SX-P14	33.560	0.284	9.531
ASTM A-283 Gr. D	Square Attachment	SX-A03	SX-P14	33.560	0.284	9.531
6061-T6	triangle	SX-A03	SX-P06	41.310	0.098	4.030
6061-T6	triangle	SX-A03	SX-P07	41.310	0.098	4.030
6061-T6	triangle	SX-A03	SX-P08	41.310	0.098	4.030
6061-T6	triangle	SX-A03	SX-P09	41.310	0.098	4.030
6061-T6		SX-A03	-	11.560	0.098	1.128
6061-T6		SX-A03	-	11.560	0.098	1.128
Gr.8.8	big nut	SX-A03	-	11.770	0.284	3.343

Gr.8.8	big nut	SX-A03	-	11.770	0.284	3.343
Gr.8.8	nut	SX-A03	-	1.190	0.284	0.338
Gr.8.8	nut	SX-A03	-	1.190	0.284	0.338
Gr.8.8	nut	SX-A03	-	1.190	0.284	0.338
Gr.8.8	nut	SX-A03	-	1.190	0.284	0.338
Gr.8.8	bolt	SX-A03	-	4.670	0.284	1.326
Gr.8.8	bolt	SX-A03	-	4.670	0.284	1.326
Gr.8.8	bolt	SX-A03	-	4.670	0.284	1.326
Gr.8.8	bolt	SX-A03	-	4.670	0.284	1.326
Gr.8.8	washer	SX-A03	-	0.330	0.284	0.094
Gr.8.8	washer	SX-A03	-	0.330	0.284	0.094
Gr.8.8	washer	SX-A03	-	0.330	0.284	0.094
Gr.8.8	washer	SX-A03	-	0.330	0.284	0.094
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	screw	N/A	-	0.370	0.284	0.105
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer-lock	N/A	-	0.060	0.284	0.017
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
Gr.8.8	washer	N/A	-	0.040	0.284	0.011
G40.21	legs	SX-A03	SX-P04	164.100	0.284	46.539
G40.21	legs	SX-A03	SX-P05	164.100	0.284	46.539
G40.21	legs	SX-A03	SX-P05	164.100	0.284	46.539
G40.21	legs	SX-A03	SX-P04	164.100	0.284	46.539

G40.21	Main Horizontal Shaft_Option1	SX-A03	SX-P15	227.490	0.284	64.517
C12L14	little shaft_Option1	SX-A03	SX-P13	12.150	0.284	3.451
ASTM A-283 Gr. D	Square Attachment_Option1	SX-A03	SX-P14	25.510	0.284	7.245
ASTM A-283 Gr. D	Square Attachment_Option1	SX-A03	SX-P14	25.510	0.284	7.245
ASTM A-283 Gr. D	Vertical Shaft_Option1	SX-A03	SX-P13	42.360	0.284	12.030

E. Bibliography

AZO Material. (2015). *AISI 12L14 Carbon Steel (UNS G12144)*. Retrieved from AZO Material: http://www.azom.com/article.aspx?ArticleID=6604

Budynas, R. G., & Nisbett, J. K. (2011). *Shigley's Mechanical Engineering Design.* New York, NY: The McGraw-Hill Companies.

eFunda. (2015). *efunda: Stress Concentration Factor Calculator*. Retrieved from efunda.com: http://www.efunda.com/formulae/solid_mechanics/stress_concentration/calc_bb_t.cfm

Henan Gang Iron and Steel Co. Ltd. (2015). *A283 Grade D,A283GrD, A283Gr.D,A283 Grade D steel,A283 Gr D steel plate*. Retrieved from Steelspecs.com: http://www.steelspecs.com/ASTM-A283/A283GradeD-A283GrD-A283Gr.D-A283-Grade-D-steel-A283-Gr-D-steel-plate.html

Metal Suppliers Online. (2015). *A283 Grade C Carbon Steels Material Property Data Sheet*. Retrieved from SuppliersOnline.com: http://www.suppliersonline.com/propertypages/A283C.asp

Nolimits Exchange. (2015). *G-Force Lesson*. Retrieved from nolimits-exchange.com: http://nolimits-exchange.com/news/g-force-lesson/35

Pilkey, W. D., & Pilkey, D. F. (2007). *Peterson's Stress Concentration Factor, Third Edition.* Hoboken, NJ, USA: John Wiley & Sons, Inc.

Timken. (2015). *Timken Tapered Roller Bearing Catalog.* Retrieved from Timken.com: http://catalog.timken.com/app.php?RelId=6.4.7.5&bookcode=trb12

Tong, L., Xu, G., Liu, Y., Yan, D., & Zhao, X.-L. (2015). Finite element analysis and formulae for stress concentration factors of diamond bird-beak SHS T-joints. *Thin Wall Structures*, 108-120.

WebCivil. (2015). *Steel Properties*. Retrieved from WebCivil.com: http://www.webcivil.com/frmsteelproperty.aspx